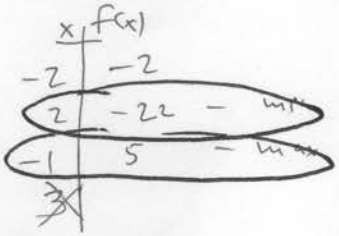
Math 140
Test 3
11.17.17

CWID: KEY

1. **Find the absolute extrema for $f(x) = x^3 - 3x^2 - 9x$ on $[-2,2]$.**

$f'(x) = 3x^2 - 6x - 9 = 3(x^2 - 2x - 3) = 3(x-3)(x+1)$

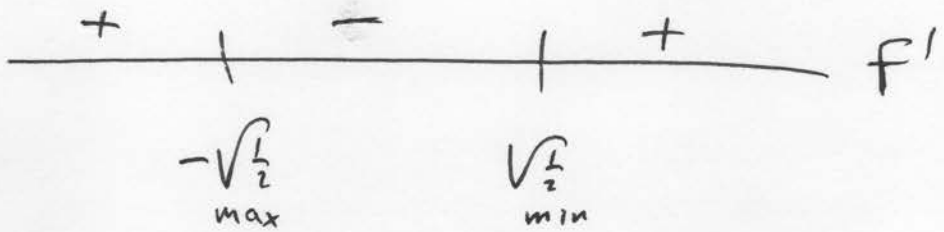
Critical values:
$x = 3, -1$



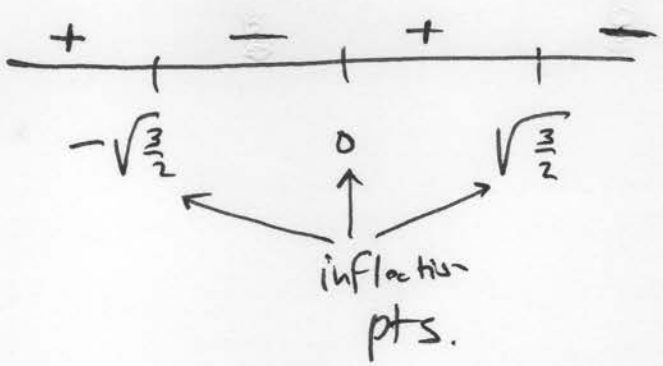
2. **Describe (but do NOT graph) the local extrema and inflection points for**

$$\mathbf{f(x) = -xe^{-x^2}}$$

$f' = -e^{-x^2}(1-2x^2)$



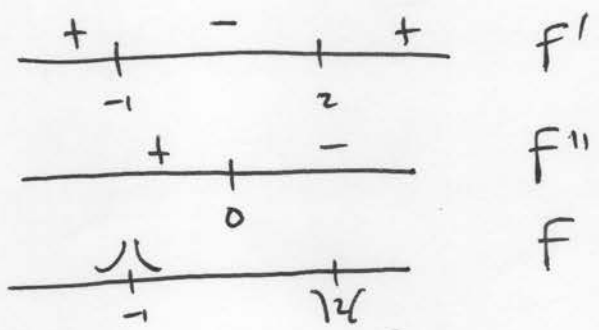
$f'' = 2x\,e^{-x^2}(3-2x^2)$

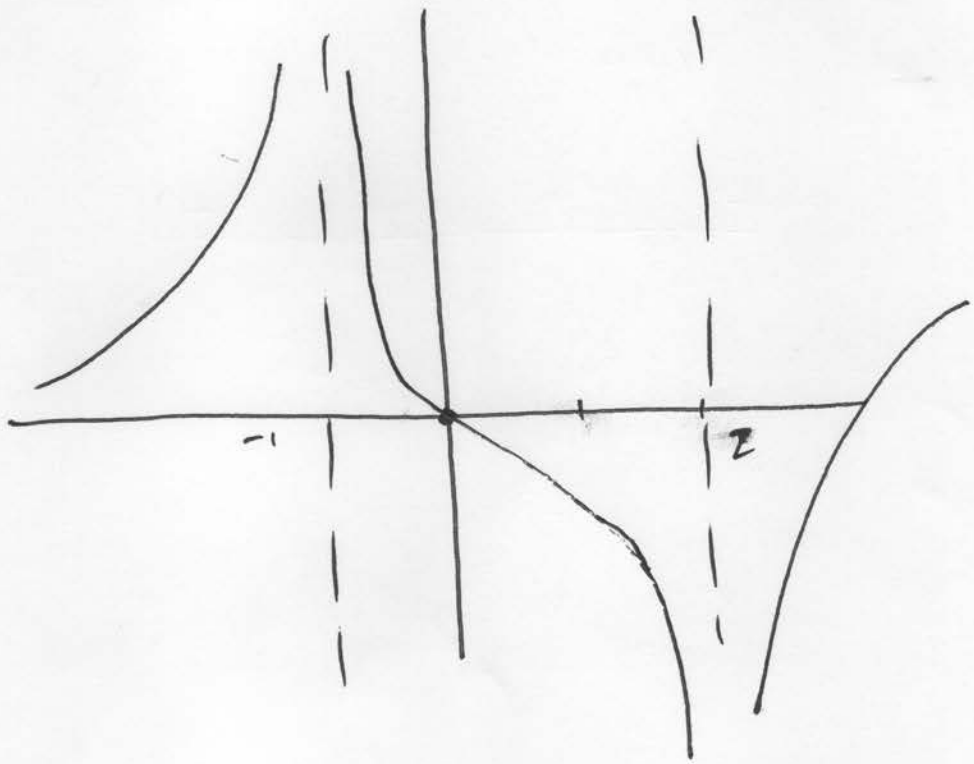


3. **Sketch the graph of f(x) if f(0) = 0, f'(0) = 0, f'(x) > 0 for x < -1 and x > 2, f'(x) < 0 for -1 < x < 2, f''(x) > 0 for x < 0, f''(x) < 0 for 0 < x, lim f(x) = +∞, lim f(x) = -∞**

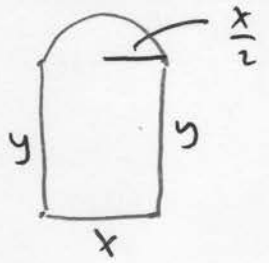
$x \to -1$

$x \to 2$





4. **Find the objective function and constraint needed to solve the following problem. Do NOT finish the optimization. A window whose top is a semicircle and bottom is a rectangle (see below) is to be constructed so that the total area is 10 and whose perimeter is as small as possible.**



$$10 = xy + \frac{1}{2}\pi\left(\frac{x}{2}\right)^2 = xy + \frac{\pi x^2}{8}$$

$$P = x + 2y + \frac{1}{2}2\pi\left(\frac{x}{2}\right) = x + 2y + \frac{\pi x}{2}$$

5. **Suppose the demand equation is given by $p\,e^{0.3q} = 400$.**
   a. **Find the elasticity.**
   b. **At what exact price will revenue be maximized?**

a) $D(q) = 400e^{-.3q}$ $\quad E = \frac{-D}{qD'} = \frac{-400e^{-.3q}}{q(-400 \cdot 0.3e^{-.3q})}$

$= \frac{1}{0.3q}$

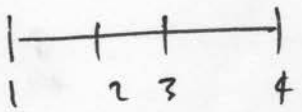
b) $q = \frac{10}{3}$ so

$p = 400e^{-1} = 147.15$

6. Find the following sums for $f(x) = (\ln x)(x^2+1)$ on $[1, 4]$. Write your answer so that **only** arithmetic remains. Do not simplify.

   a. L_3

   b. R_3



x	f(x)
1	0
2	$(\ln 2)5 = 3.46$
3	$(\ln 3)10 = 10.98$
4	$(\ln 4)17 = 23.56$

a) $1(0 + 3.46 + 10.98) = 14.44$

b) $1(3.46 + 10.98 + 23.56) = 38$

7. **Evaluate the following integrals. Write your answers so that only arithmetic remains. Do not simplify.**

a. $\int \left(1 - \frac{1}{x} + \frac{1}{x^2} - \frac{1}{x^3}\right) dx$

b. $\int_1^2 \left(\frac{4}{x} + e^{-3x} - \sqrt{x}\right) dx$

a) $x - \ln x + \frac{x^{-1}}{-1} - \frac{x^{-2}}{-2} + C$

b) $\left(4\ln x + \frac{e^{-3x}}{-3} - \frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right)\Big|_1^2 = 1.569$