Math 140
Test 3
11.17.17

CWID: KEY

1. **Find the absolute extrema for $f(x) = 2x^3+3x^2-12x$ on $[-3,3]$.**

$f'(x) = 6x^2 + 6x - 12 = 6(x^2 + x - 2)$

$= 6(x+2)(x-1)$ critical values: $x = -2, 1$

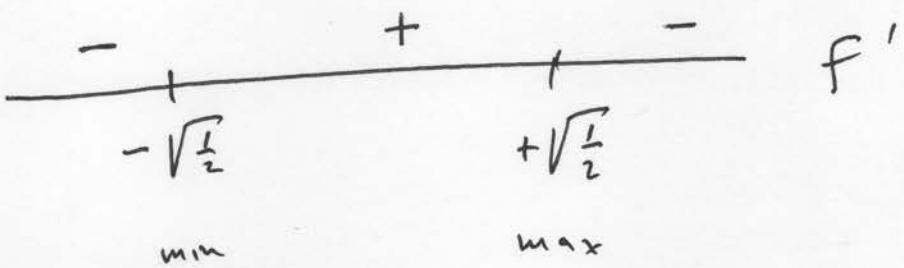
x	$f(x)$	
-3	9	
3	45	MAX
-2	20	
1	-7	MIN

2. **Describe (but do NOT graph) the local extrema and inflection points for**

$$\mathbf{f(x) = xe^{-x^2}}$$

$$f'(x) = x(-2xe^{-x^2}) + e^{-x^2}$$

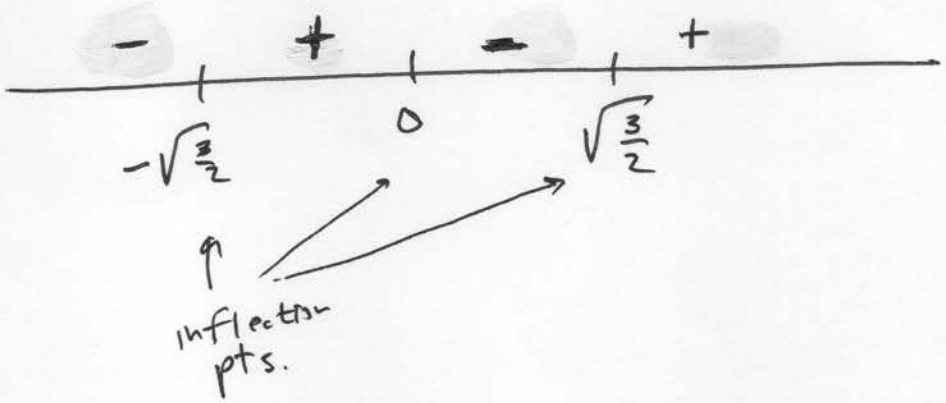
$$= e^{-x^2}(1-2x^2)$$



$$f''(x) = e^{-x^2}(-4x) + (1-2x^2)(-2xe^{-x^2})$$

$$= -2x\,e^{-x^2}(2 + 1 - 2x^2)$$

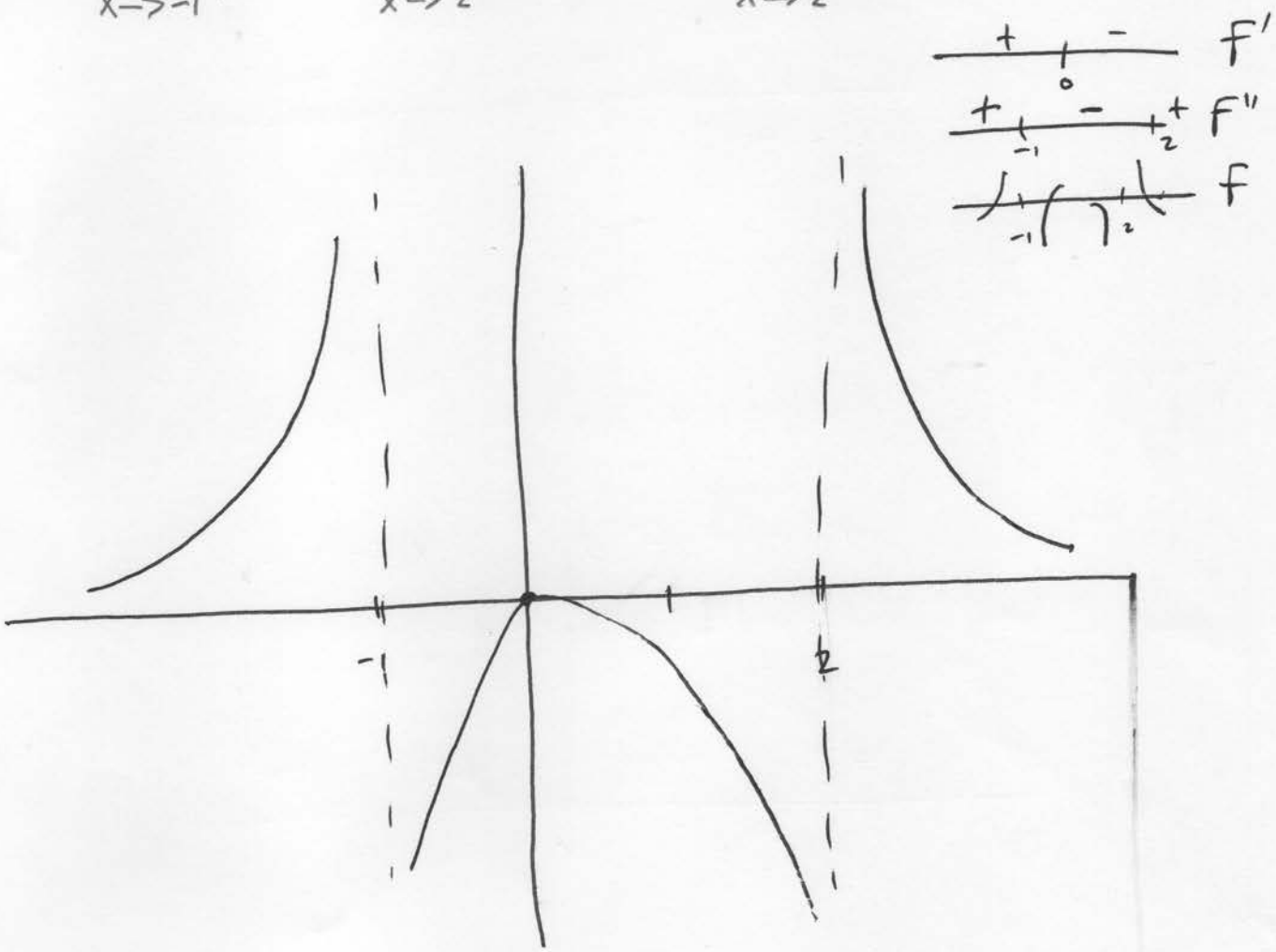
$$= -2x\,e^{-x^2}(3 - 2x^2)$$



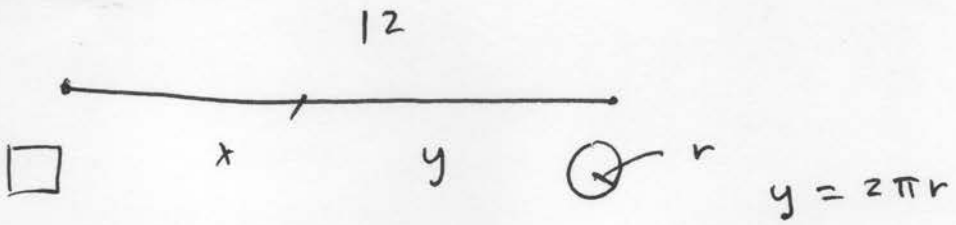
3. **Sketch the graph of f(x) if f(0) = 0, f'(0) = 0, f'(x) > 0 for x < 0, f'(x) < 0 for x > 0, f''(x) > 0 for x < -1 and x > 2, f''(x) < 0 for -1 < x < 2, f'(-1) & f'(2) DNE,**

$\lim_{x \to -1^-} f(x) = +\infty$,

$\lim_{x \to -1^+} f(x) = -\infty$, $\lim_{x \to 2^-} f(x) = -\infty$, and $\lim_{x \to 2^+} f(x) = +\infty$



4. **Find the objective function and constraint needed to solve the following problem. Do NOT finish the optimization. A wire 12 cm long is to be cut into two pieces so that one piece forms a circle and the other forms a square. The sum of the areas is to be as small as possible.**



$y = 2\pi r$

Constraint $\quad x + y = 12$

Objective $\quad A = \left(\frac{x}{4}\right)^2 + \pi\left(\frac{y}{2\pi}\right)^2$

5. **Suppose the demand equation is given by $p\,e^{0.1q} = 200$.**
   a. **Find the elasticity.**
   b. **At what exact price will revenue be maximized?**

$$p = D(q) = 200\,e^{-0.1q}$$

a) $$E = \frac{-D}{qD'} = \frac{-200\,e^{-0.1q}}{q\,(-200 \cdot 0.1\,e^{-0.1q})}$$

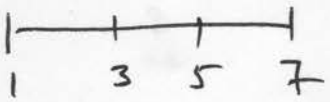
$$= \frac{1}{0.1q} = \frac{10}{q}$$

b) $q = 10$ so $p = 200\,e^{-1} = 73.58$

6. **Find the following sums for $f(x) = (\ln x)/(e^x)$ on [1, 7].**
Write your answer so that only arithmetic remains. Do not simplify.

a. L_3

b. R_3



x	f(x)
1	0
3	$\ln 3/e^3 = 0.05$
5	$\ln 5/e^5 = 0.01$
7	$\ln 7/e^7 = 0.017$

a) $2(0 + .05 + .01) = (.06)2 = .12$

b) $2(0.05 + .01 + .017) = 2(0.077) = 0.154$

7. **Evaluate the following integrals. Write your answers so that only arithmetic remains. Do not simplify.**

a. $\int_1^3 \left(\frac{4}{x} + \sqrt{x} + e^{-2x}\right) dx$

b. $\int \left(1 + x + x^2 + x^3\right) dx$

a) $\left(4\ln x + \frac{2}{3}x^{\frac{3}{2}} + \frac{e^{-2x}}{-2}\right)\Big|_1^3 = 7.25831$

b) $x + \frac{x^2}{2} + \frac{x^3}{3} + \frac{x^4}{4} + C$