Math 140
Test 3
3.29.19

CWID: KEY

1. Find the equation of the tangent line to $x^2 + xy^3 + e^{xy} = 2$ at (1,0) using implicit differentiation.

$2x + y^3 + 3xy^2y' + e^{xy}(xy' + y) = 0$

let $x = 1$, $y = 0$

$2 + 0 + 0 + e^0(y' + 0) = 0$

so $y' = -2$

line: $y - 0 = -2(x - 1)$ or

$y = -2x + 2$

2. **A pecan pie comes out of a 400° oven at noon and is placed in a room whose temperature is 70°. At 1:00 p.m. the pie has cooled to 360°. What time will it be when the pie is 300°?**

t	T
0	400
1	360

$T = T_a + (T_0 - T_a)e^{-kt}$

$T_a = 70, \; T_0 = 400$

$360 = 70 + 330e^{-k}$ $\qquad k = -\ln\left(\frac{290}{330}\right) = 0.129$

$300 = 70 + 330e^{-.129t}$

$$t = \frac{\ln\left(\frac{230}{330}\right)}{-.129} = 2.799 \text{ hours after noon}$$

or 2:48 pm

3. **Find the absolute extrema for $f(x) = 2x^3+3x^2-12x$ on [-3,0].**

$f' = 6x^2 + 6x - 12 = 6(x^2+x-2)$

$= 6(x+2)(x-1)$

critical values are $x = -2, +1$

So $x = -2$ only

x	y	
-3	9	
0	0	Abs min
-2	20	Abs max

4. **Use sign charts to describe (but NOT graph) the local extrema and inflection points for $f(x) = x^2e^{-x}$.**

$f' = x^2(-e^{-x}) + 2xe^{-x} = e^{-x}(2x - x^2) = xe^{-x}(2-x)$

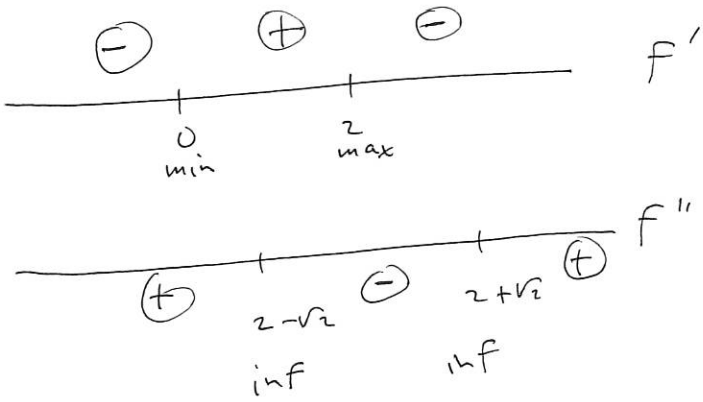
$f'' = e^{-x}(2-2x) - e^{-x}(2x - x^2) = e^{-x}(2 - 2x - 2x + x^2)$

$= e^{-x}(x^2 - 4x + 2)$

$= 0$ when

$x = \frac{4 \pm \sqrt{8}}{2}$

$= 2 \pm \sqrt{2}$

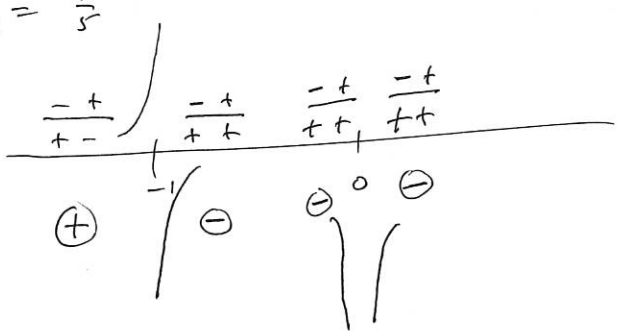


5. **Give a complete description of the vertical and horizontal asymptotic behavior of the following function, but do NOT sketch the graph:**

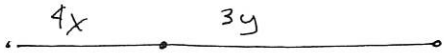
$$y = \frac{(x-2)^3(x-1)^2}{5x^2(x+1)^3}$$

HA $y = \frac{1}{5}$

VA

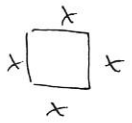


6. Find the objective function and constraint for the following optimization problem. DO NOT FINISH THE PROBLEM. THIS IS SET UP ONLY. ***A wire of length 10 cm is cut into two pieces. One piece is bent to form a square, the other to form an equilateral triangle. Determine where to place the cut so that the sum of the areas of the square and triangle is to be as small as possible.***



constraint $4x + 3y = 10$

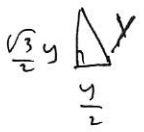
objective $A = x^2 + \frac{\sqrt{3}y^2}{4}$



$\downarrow$

x^2

$\downarrow$



$2 \cdot \frac{1}{2} \cdot \frac{\sqrt{3}}{2} \cdot y \cdot \frac{y}{2}$

$= \frac{\sqrt{3}y^2}{4}$

7. The price, p, is related to the demand, q, by the following equation: $pe^{0.04q} = 200$.
   a. Find the elasticity E, in general, for this scenario.
   b. Find the price that will yield maximum revenue.
   c. What price range will yield a value of E that is elastic?

$p = 200e^{-.04q} = D(q)$

a) $E = \frac{-D}{qD'} = \frac{-200e^{-.04q}}{q \cdot 200e^{-.04q}(-.04)} = \frac{1}{.04q}$

b) $q = 25 \rightarrow p = 200e^{-.04(25)} = \frac{200}{e} = 73.579$

c) $q < 25$ so $p > 73.579$