Math 141
Test 1
9.23.11

GREEN

CWID: KEY

SHOW YOUR WORK.

1. Let $f(x,y) = (e^{6x} + x^2)(e^{3y} + 5)$. Find f_x.

$f_x = (e^{3y} + 5)(6e^{6x} + 2x)$

2. Find and classify all relative maxima and minima for $f(x,y) = -x^2 - 8xy - y^2$ by using the second derivative test.

$f_x = -2x - 8y = 0 \rightarrow x = -4y$

$f_y = -8x - 2y = 0 \quad -8(-4y) - 2y = 0$

$30y = 0$

$x = y = 0$ is the only possible min/max

$f_{xx} = -2 \qquad f_{yy} = -2$

$f_{xy} = -8$

$D = (-2)(-2) - (-8)^2 < 0$

So $x = y = 0$ is neither a min or max.

3. Use the method of Lagrange Multipliers to find the location of the relative maximum if $F(x,y,\lambda) = x^2 - y^2 + \lambda(2x + y - 3)$.

$F_x = 2x + 2\lambda = 0 \quad \lambda = -x$

$F_y = -2y + \lambda = 0 \quad \lambda = 2y$ $\Big\}$ $x = -2y$

$F_\lambda = 2x + y - 3 = 0 \longrightarrow -4y + y - 3 = 0$

$-3y - 3 = 0$

$y = -1, \; x = 2$

4. Find the equation of the Least Squares Fit Line for the following data:

	x	y	xy	x^2
	0	1	0	0
	1	-1	-1	1
	2	2	4	4
	3	1	3	9
Σ	6	3	6	14

$N = 4$

$y = Ax + B$

$$A = \frac{4 \cdot 6 - 6 \cdot 3}{4 \cdot 14 - 6 \cdot 6} = \frac{6}{20} = .3$$

$$B = \frac{3}{4} - .3\,\frac{6}{4} = \frac{1.2}{4} = .3$$

$$y = .3x + .3$$

5. Solve the following system by row-reducing the augmented matrix:

$$x - y = 10$$
$$2x + y = 15$$

$$\begin{pmatrix} 1 & -1 & 10 \\ 2 & 1 & 15 \end{pmatrix}$$

$$\rightarrow \begin{pmatrix} 1 & -1 & 10 \\ 0 & 3 & -5 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & -1 & 10 \\ 0 & 1 & -\frac{5}{3} \end{pmatrix}$$

$$\rightarrow \begin{pmatrix} 1 & 0 & \frac{25}{3} \\ 0 & 1 & -\frac{5}{3} \end{pmatrix}$$

$$x = \frac{25}{3}$$
$$y = -\frac{5}{3}$$

6. Let the price (R) and production (P) matrices for three factories (a, b, and c), each of which manufactures two products (p_1 and p_2), be given by:

$$R = \begin{matrix} p_1 & p_2 \end{matrix} \begin{bmatrix} \$20 & \$10 \end{bmatrix}, \quad P = \begin{matrix} \\ p_1 \\ p_2 \end{matrix} \begin{matrix} a & b & c \end{matrix} \begin{bmatrix} 2 & 6 & 0 \\ 4 & 2 & 1 \end{bmatrix}$$

Calculate RP and interpret the results.

$$RP = \begin{pmatrix} 20 & 10 \end{pmatrix} \begin{pmatrix} 2 & 6 & 0 \\ 4 & 2 & 1 \end{pmatrix} = \begin{pmatrix} 80 & 140 & 10 \end{pmatrix}$$

Revenue from the 3 factories are 80, 140, and 10.

7. Find A^{-1} and then use that inverse to solve the following system:

$$A = \begin{bmatrix} 1 & 1 & -2 \\ 0 & 0 & 1 \\ 1 & 2 & 0 \end{bmatrix} \qquad AX = \begin{bmatrix} 3 \\ -4 \\ 2 \end{bmatrix}$$

$$\left(\begin{array}{ccc|ccc} 1 & 1 & -2 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 1 & 2 & 0 & 0 & 0 & 1 \end{array}\right) \rightarrow \left(\begin{array}{ccc|ccc} 1 & 1 & -2 & 1 & 0 & 0 \\ 1 & 2 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 \end{array}\right)$$

$$\rightarrow \left(\begin{array}{ccc|ccc} 1 & 1 & -2 & 1 & 0 & 0 \\ 0 & 1 & 2 & -1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 \end{array}\right) \rightarrow \left(\begin{array}{ccc|ccc} 1 & 0 & -4 & 2 & 0 & -1 \\ 0 & 1 & 2 & -1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 \end{array}\right)$$

$$\rightarrow \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 2 & 4 & -1 \\ 0 & 1 & 0 & -1 & -2 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 \end{array}\right) \rightarrow A^{-1}$$

$$X = A^{-1}\begin{pmatrix} 3 \\ -4 \\ 2 \end{pmatrix} = \begin{pmatrix} 2 & 4 & -1 \\ -1 & -2 & 1 \\ 0 & 1 & 0 \end{pmatrix}\begin{pmatrix} 3 \\ -4 \\ 2 \end{pmatrix} = \begin{pmatrix} -12 \\ 7 \\ -4 \end{pmatrix}$$

8. A two sector economy is based on producing nuclear and geothermal power. In order to produce \$1 of nuclear energy, \$0.20 of nuclear and \$0.10 of geothermal are required; in order to produce \$1 of geothermal energy, \$0.50 of nuclear and \$0.10 of geothermal are required. How much nuclear and geothermal energy should be produced in order to meet a demand of \$20 of nuclear and \$10 of geothermal energy?

$$X = (I-A)^{-1}D = \left(\begin{pmatrix}1&0\\0&1\end{pmatrix} - \begin{pmatrix}.2&.5\\.1&.1\end{pmatrix}\right)^{-1}\begin{pmatrix}20\\10\end{pmatrix}$$

$$= \begin{pmatrix}.8&-.5\\-.1&.9\end{pmatrix}^{-1}\begin{pmatrix}20\\10\end{pmatrix}$$

$$= \frac{1}{.8\cdot.9-.1\cdot.5}\begin{pmatrix}.9&.5\\.1&.8\end{pmatrix}\begin{pmatrix}20\\10\end{pmatrix}$$

$$= \frac{100}{67}\begin{pmatrix}23\\10\end{pmatrix} = \begin{pmatrix}34.33\\14.93\end{pmatrix}$$