Math 141
Test 1
9.23.11

ORANGE

CWID: KEY

SHOW YOUR WORK.

1. Let $f(x,y) = (x^2)(1 + e^{3y})$. Find f_y.

$$f_y = x^2(3e^{3y})$$

2. Find and classify all relative maxima and minima for $f(x,y) = -2x^2 + 2xy - y^2 + 4x - 6y + 5$ by using the second derivative test.

$$f_x = -4x + 2y + 4 = 0$$

$$f_y = 2x - 2y - 6 = 0$$

$$y = 2x - 2$$

$$2x - 2(2x-2) - 6 = 0$$

$$-2x + 4 - 6 = 0$$

$$x = -1 \ , \ y = -4$$

$$f_{xx} = -4 \qquad f_{xy} = 2$$

$$f_{yy} = -2$$

$$D = 8 - 2^2 > 0$$

$$f_{xx} < 0$$

So we have a relative maximum

at $(-1, -4)$

3. Use the method of Lagrange Multipliers to find the location of the relative minimum if $F(x,y,\lambda) = x^2 + 3y^2 + 10 + \lambda(8 - x - y)$.

$F_x = 2x - \lambda$

$F_y = 6y - \lambda$

$F_\lambda = 8 - x - y$

$\lambda = 2x = 6y \rightarrow x = 3y$

$0 = 8 - 3y - y$

$y = 2,\ x = 6$

4. Find the equation of the Least Squares Fit Line for the following data:

	x	y	xy	x^2
	0	1	0	0
	1	-1	-1	1
	2	-2	-4	4
	3	-1	-3	9
Σ	6	-3	-8	14

$y = Ax + B$

$N = 4$

$$A = \frac{4\cdot(-8) - 6(-3)}{4\cdot 14 - 6^2} = \frac{-32+18}{56-36} = \frac{-14}{20} = -.7$$

$$B = -\frac{3}{4} + .7\,\frac{6}{4} = +\frac{1.2}{4} = .3$$

$$y = -.7x + .3$$

5. Solve the following system by row-reducing the augmented matrix:

$$\begin{aligned} x \; - \; y &= 10 \\ 2x \; - \; 2y &= 15 \end{aligned}$$

$$\begin{pmatrix} 1 & -1 & 10 \\ 2 & -2 & 15 \end{pmatrix}$$

$$\to \begin{pmatrix} 1 & -1 & 10 \\ 0 & 0 & -5 \end{pmatrix}$$

NO <u>SOLUTION</u>

6. Let the price (R) and production (P) matrices for three factories (a, b, and c), each of which manufactures two products (p_1 and p_2), be given by:

$$R = \begin{array}{cc} p_1 & p_2 \\ [\$10 & \$20] \end{array}, \quad P = \begin{array}{c|ccc} & a & b & c \\ p_1 & 2 & 5 & 0 \\ p_2 & 4 & 3 & 1 \end{array}$$

Calculate RP and interpret the results.

$RP = (100 \quad 110 \quad 20)$

Factory Revenues are 100, 110, + 20.

7. Find A^{-1} and then use that inverse to solve the following system:

$$A = \begin{bmatrix} 0 & 0 & 1, \\ 1 & 2 & 1 \\ 1 & 1 & -2 \end{bmatrix} \qquad AX = \begin{bmatrix} 3 \\ 4 \\ 2 \end{bmatrix}$$

$$\left(\begin{array}{ccc|ccc} 0 & 0 & 1 & 1 & 0 & 0 \\ 1 & 2 & 1 & 0 & 1 & 0 \\ 1 & 1 & -2 & 0 & 0 & 1 \end{array}\right) \to \left(\begin{array}{cccccc} 1 & 1 & -2 & 0 & 0 & 1 \\ 1 & 2 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \end{array}\right) \to \left(\begin{array}{cccccc} 1 & 1 & -2 & 0 & 0 & 1 \\ 0 & 1 & 3 & 0 & 1 & -1 \\ 0 & 0 & 1 & 1 & 0 & 0 \end{array}\right)$$

$$\to \left(\begin{array}{cccccc} 1 & 0 & -5 & 0 & -1 & 2 \\ 0 & 1 & 3 & 0 & 1 & -1 \\ 0 & 0 & 1 & 1 & 0 & 0 \end{array}\right) \to \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 5 & -1 & 2 \\ 0 & 1 & 0 & -3 & 1 & -1 \\ 0 & 0 & 1 & 1 & 0 & 0 \end{array}\right)$$

$$X = \begin{pmatrix} 5 & -1 & 2 \\ -3 & 1 & -1 \\ 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} 3 \\ 4 \\ 2 \end{pmatrix} = \begin{pmatrix} 15 \\ -7 \\ 3 \end{pmatrix}$$

8. A two sector economy is based on producing nuclear and geothermal power. In order to produce \$1 of nuclear energy, \$0.10 of nuclear and \$0.20 of geothermal are required; in order to produce \$1 of geothermal energy, \$0.40 of nuclear and \$0.30 of geothermal are required. How much nuclear and geothermal energy should be produced in order to meet a demand of \$10 of nuclear and \$20 of geothermal energy?

$$X = \left(\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} - \begin{pmatrix} .1 & .4 \\ .2 & .3 \end{pmatrix} \right)^{-1} \begin{pmatrix} 10 \\ 20 \end{pmatrix}$$

$$= \begin{pmatrix} .9 & -.4 \\ -.2 & .7 \end{pmatrix}^{-1} \begin{pmatrix} 10 \\ 20 \end{pmatrix}$$

$$= \frac{1}{.63 - .08} \begin{pmatrix} .7 & .4 \\ .2 & .9 \end{pmatrix} \begin{pmatrix} 10 \\ 20 \end{pmatrix}$$

$$= \frac{100}{55} \begin{pmatrix} 15 \\ 20 \end{pmatrix} = \begin{pmatrix} 27.27 \\ 36.36 \end{pmatrix}$$