Math 141

Test 1

9.23.11

CWID: KEY

SHOW YOUR WORK.

1. Let $f(x,y) = (y^2 + 1)(e^{6x} + e^{4y})$. Find f_y.

$$f_y = (y^2+1)(4e^{4y}) + (e^{6x}+e^{4y})(2y)$$

2. Find and classify all relative maxima and minima for $f(x,y) = x^2 + 2xy + 5y^2 + 2x + 10y - 3$ by using the second derivative test.

$f_x = 2x + 2y + 2 = 0 \quad x = -y - 1$

$f_y = 2x + 10y + 10 = 0 \quad 2(-y-1) + 10y + 10 = 0$

$8y + 8 = 0 \quad y = -1, x = 0$

$f_{xx} = 2 \quad f_{yy} = 10 \quad f_{xy} = 2$

$D = 2 \cdot 10 - 4 > 0$

$f_{xx} = 2 > 0$

relative minimum at $(0, -1)$

3. Use the method of Lagrange Multipliers to find the location of the relative maximum if $F(x,y,\lambda) = x^2 + xy - 3y^2 + \lambda(2 - x - 2y)$.

$F_x = 2x + y - \lambda$

$F_y = x - 6y - 2\lambda$

$F_\lambda = 2 - x - 2y$

$\lambda = 2x + y$

$\lambda = \frac{\cancel{x} - 6y}{2}$

$4x + 2y = x - 6y$

$3x = -8y$

$x = -\frac{8}{3}y$

$0 = 2 + \frac{8}{3}y - 2y = 2 + \frac{2}{3}y$

$y = -3 \quad x = 8$

4. Find the equation of the Least Squares Fit Line for the following data:

x	y	xy	x^2
0	1	0	0
-1	-1	1	1
1	2	2	1
2	-1	-2	4
Σ 2	1	1	6

$N=4 \qquad y = Ax+B$

$$A = \frac{4\cdot 1 - 2(1)}{4\cdot 6 - 2\cdot 2} = \frac{2}{20} = .1$$

$$B = \frac{1}{4} - .1\,\frac{2}{4} = \frac{.8}{4} = .2$$

$$y = .1x + .2$$

5. Solve the following system by row-reducing the augmented matrix:

$$\begin{aligned} x \; - \; y &= 10 \\ 3x \; - \; 3y &= 30 \end{aligned}$$

$$\begin{pmatrix} 1 & -1 & 10 \\ 3 & -3 & 30 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & -1 & 10 \\ 0 & 0 & 0 \end{pmatrix}$$

$y = y \quad x = 10 + y$

6. Let the price (R) and production (P) matrices for three factories (a, b, and c), each of which manufactures two products (p_1 and p_2), be given by:

$$R = \begin{array}{cc} p_1 & p_2 \\ [\$10 & \$30] \end{array}, \quad P = \begin{array}{c} \\ p_1 \\ p_2 \end{array} \begin{array}{c} \begin{array}{ccc} a & b & c \end{array} \\ \begin{bmatrix} 2 & 5 & 4 \\ 4 & 3 & 1 \end{bmatrix} \end{array}$$

Calculate RP and interpret the results.

$RP = (140 \quad 140 \quad 70)$

The 3 factories have revenues of 140, 140, and 70.

7. Find A^{-1} and then use that inverse to solve the following system:

$$A = \begin{bmatrix} 1 & -1 & 4, \\ 0 & 0 & 1 \\ -1 & 2 & 3 \end{bmatrix} \qquad AX = \begin{bmatrix} 3 \\ 4 \\ 1 \end{bmatrix}$$

$$\left(\begin{array}{ccc|ccc} 1 & -1 & 4 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ -1 & 2 & 3 & 0 & 0 & 1 \end{array}\right) \rightarrow \left(\begin{array}{cccccc} 1 & -1 & 4 & 1 & 0 & 0 \\ -1 & 2 & 3 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 \end{array}\right)$$

$$\left(\begin{array}{cccccc} 1 & -1 & 4 & 1 & 0 & 0 \\ 0 & 1 & 7 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 \end{array}\right) \rightarrow \left(\begin{array}{cccccc} 1 & 0 & 11 & 2 & 0 & 1 \\ 0 & 1 & 7 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 \end{array}\right)$$

$$\left(\begin{array}{cccccc} 1 & 0 & 0 & 2 & -11 & 1 \\ 0 & 1 & 0 & 1 & -7 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 \end{array}\right)$$

$$X = \begin{pmatrix} 2 & -11 & 1 \\ 1 & -7 & 1 \\ 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 3 \\ 4 \\ 1 \end{pmatrix} = \begin{pmatrix} -37 \\ -24 \\ 4 \end{pmatrix}$$

8. A two sector economy is based on producing nuclear and geothermal power. In order to produce \$1 of nuclear energy, \$0.20 of nuclear and \$0.30 of geothermal are required; in order to produce \$1 of geothermal energy, \$0.40 of nuclear and \$0.20 of geothermal are required. How much nuclear and geothermal energy should be produced in order to meet a demand of \$5 of nuclear and \$20 of geothermal energy?

$$X = \left(\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} - \begin{pmatrix} .2 & .4 \\ .3 & .2 \end{pmatrix} \right)^{-1} \begin{pmatrix} 5 \\ 20 \end{pmatrix} = \begin{pmatrix} .8 & -.4 \\ -.3 & .8 \end{pmatrix}^{-1} \begin{pmatrix} 5 \\ 20 \end{pmatrix}$$

$$= \frac{1}{.64 - .12} \begin{pmatrix} .8 & .4 \\ .3 & .8 \end{pmatrix} \begin{pmatrix} 5 \\ 20 \end{pmatrix} = \frac{100}{52} \begin{pmatrix} 12 \\ 17.5 \end{pmatrix}$$

$$= \begin{pmatrix} 23.08 \\ 33.07 \end{pmatrix}$$