Math 141^{c}

Test 3

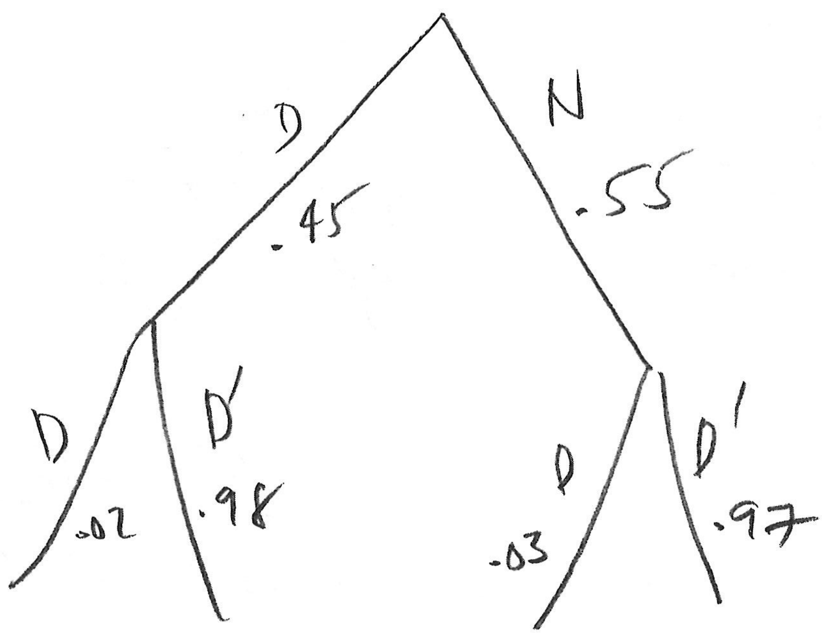
November 18, 2011

CWID:____________

BLUE

1. The quality control division of GM is interested in analyzing the quality of their cars as manufactured in two locations: Detroit and Nashville. They discover that 2% of the cars manufactured in Detroit are defective, while 3% of those made in Nashville are defective. GM produces 45% of its cars in Detroit and 55% in Nashville.
   a. Draw a tree diagram for this problem.
   b. What is the probability that a car, chosen at random, will be free of defects?

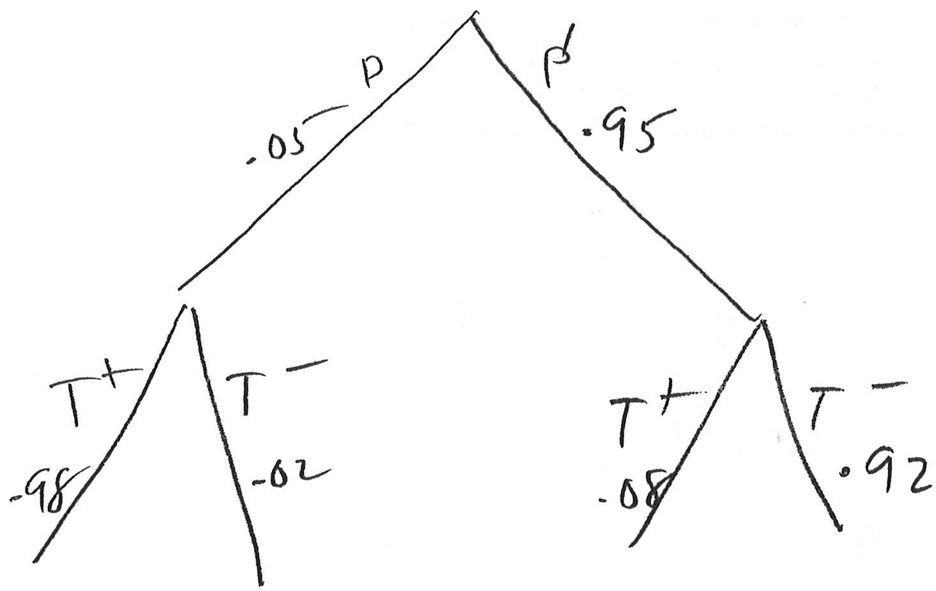
a)



b) $P(D') = .45 \times .98 + .55 \times .97$

$= .9745$

2. A young lady wishes to determine whether she is pregnant, so she buys a *ClearBlue* pregnancy test kit at her local drugstore. The kit is 98% accurate if the young lady is pregnant, and 92% accurate if they are not pregnant. Assume that generally, such a young lady is 5% likely to be pregnant. Suppose that this young lady administers her *ClearBlue* test and it indicates that she is pregnant. What is the probability that she actually is pregnant?

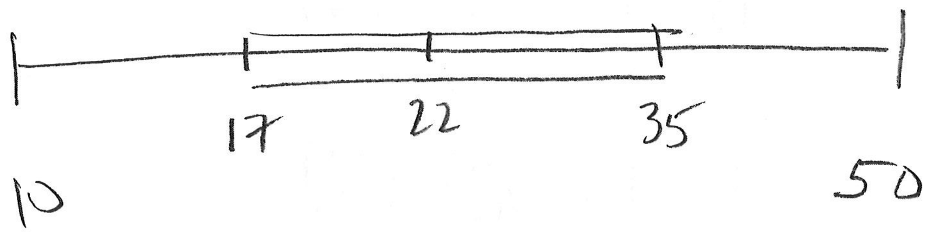


$$P(P \mid T^+) = \frac{.05 \times .98}{.05 \times .98 + .95 \times .08}$$

$$= .392$$

3. Draw the Box & Whisker Plot for the following data:
{15, 17, 22, 17, 35, 23, 10, 50, 22, 50, 23}

10 15 17 17 22 22 23 23 35 50 50



4. An urn contains 25 red marbles and 12 black marbles. Three marbles are drawn, without replacement. Let X be the number of red marbles drawn. Find the probability distribution for X.

x	$p(x)$
0	$\binom{12}{3} / \binom{37}{3}$
1	$\binom{25}{1}\binom{12}{2} / \binom{37}{3}$
2	$\binom{25}{2}\binom{12}{1} / \binom{37}{3}$
3	$\binom{25}{3} / \binom{37}{3}$

5. A poll shows that 20% percent of the voters in a certain Texas county are members of the Tea Party. A survey of fifteen randomly chosen voters from this county is conducted. Determine the **exact numeric values** of the following: $X \sim Bin(15, .2)$

   a. The expected number of voters from the survey who are NOT Tea Party members.

   b. The probability that at least two of the surveyed voters are Tea Party members.

a) $\mu = nq = 15 \times .8 = 12$

b) $P(X \geq 2) = 1 - P(X=0) - P(X=1)$

$= 1 - \binom{15}{0} .2^0 .8^{15} - \binom{15}{1} .2^1 .8^{14}$

$= .8328$

6. State Farm offers annual health insurance to married couples with maximum coverage as follows: $50000 per year if only the husband is sick, $30000 per year if just the wife is sick, and $70000 if both wife and husband get sick. Suppose that the husband has a 15% chance of getting sick, while the wife has a 3% chance, and that these probabilities are independent of one another. What should a couple expect to pay for this insurance coverage?

X	$p(x)$
0	$.85 \times .97$
30000	$.03 \times .85$
50000	$.15 \times .97$
70000	$.15 \times .03$

$\mu = 8355$, so pay $8355

7. College students average 5 hours of sleep per night with a standard deviation of 0.25 hours. Out of 1000 students, approximately how many of them will sleep between 4.5 and 5.5 hours?

Chebychev $\mu = 5$, $\sigma = .25$

$P(4.5 < X < 5.5) = P(\mu - .5 < X \le \mu + .5)$

$\ge 1 - \frac{(.25)^2}{(.5)^2} = .75$

So except 750 students.

8. The lifespan of a set of tires is distributed as X ~ N(28, 4), where X is measured in months. Find the probability that a randomly chosen set of tires lasts:
   a. more than 26 months.
   b. between 25 and 27 months.

a) $P(X > 26) = P\left(Z > \frac{26-28}{2}\right)$

$= P(Z > -1) = 1 - .1587$

$= .8413$

b) $P(25 < X < 27)$

$= P\left(\frac{25-28}{2} < Z < \frac{27-28}{2}\right)$

$= P(-1.5 < Z < -0.5)$

$= .2417$

9. Every time Susan attends mathematics class, she has a 20% chance of falling asleep. Suppose she attends 30 classes. **Approximate** the following probabilities, leaving your answers in decimal form:
   a. Susan falls asleep more than 5 times.
   b. Susan falls asleep between 7 and 9 times, inclusive.

a) .5793

b) .3659

10. BONUS QUESTION – 8 points
Adohr Farms sells bottles of milk that are guaranteed to stay fresh for up to 15 days, with the chance of spoilage during this period being 4%. A consumer wants to buy enough milk to be sure to have at least a 99.9% chance of having at least two good bottles of milk on hand at the end of day 15. How many bottles should he buy?

X = # good bottles $\sim Bin(n, .96)$

$n = ?$

$P(X \geq 2) \geq .999$

$1 - P(X \leq 1) \geq .999$

$P(X \leq 1) \leq .001$

$P(X = 0) + P(X = 1) \leq .001$

$\binom{n}{0} .96^0 .04^n + \binom{n}{1} .96^1 .04^{n-1} \leq .001$

n	$P(X \leq 1)$
2	.0784
3	.004672
4	.000248
5	

$\leftarrow$ 4 Bottles !!