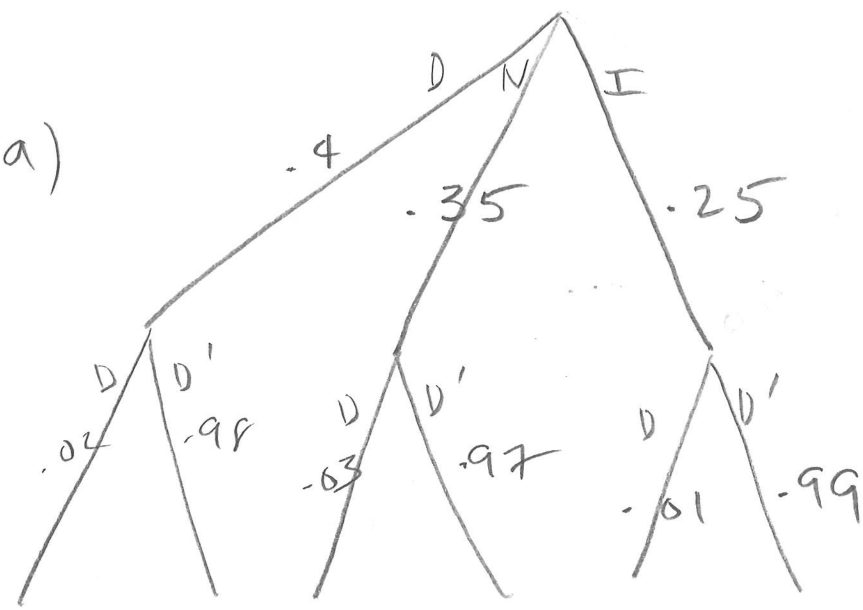
Math 141[a]
Test 3
November 18, 2011
CWID:___________

ORANGE

1. The quality control division of GM is interested in analyzing the quality of their cars as manufactured in three locations: Detroit, Nashville, and Indianapolis. They discover that 2% of the cars manufactured in Detroit are defective, 3% of those made in Nashville are defective, and 1% of the cars from Indianapolis are defective. GM produces 40% of its cars in Detroit, 35% in Nashville, and 25% in Indianapolis.
    a. Draw a tree diagram for this problem.
    b. What is the probability that a car, chosen at random, will be defective?

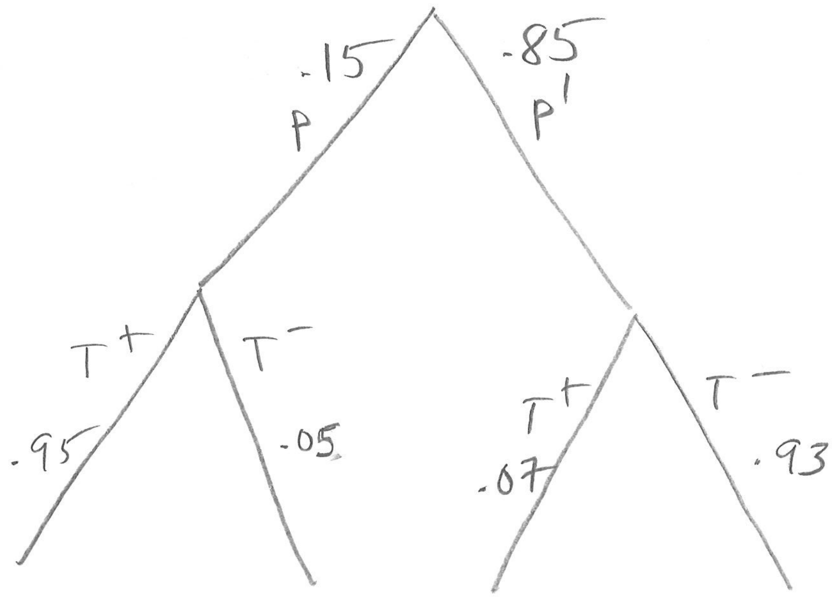
a)



b) $P(D) = .4 \times .02 + .35 \times .03 + .25 \times .01$

$= .0206$

2. A young lady wishes to determine whether she is pregnant, so she buys a *ClearBlue* pregnancy test kit at her local drugstore. The kit is 95% accurate if the young lady is pregnant, and 93% accurate if they are not pregnant. Assume that generally, such a young lady is 15% likely to be pregnant. Suppose that this young lady administers her *ClearBlue* test and it indicates that she is NOT pregnant. What is the probability that she actually is pregnant?



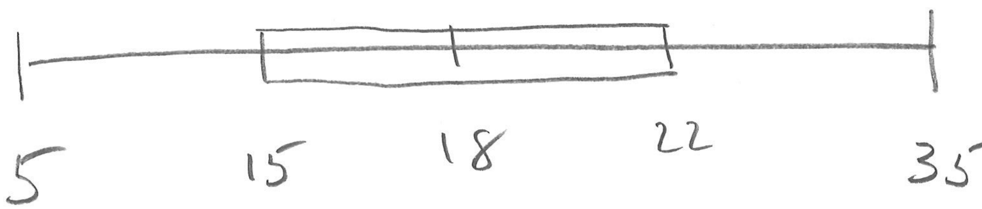
$$P(P \mid T^-) = \frac{.15 \times .05}{.15 \times .05 + .85 \times .93}$$

$$= .0094$$

3. Draw the Box & Whisker Plot for the following data: {15, 19, 22, 17, 35, 17, 23, 10, 5, 22}

5 10 15 17 17 19 22 22 23 35

18



4. An urn contains 5 red marbles and 10 black marbles. Three marbles are drawn, without replacement. Let X be the number of black marbles drawn. Find the probability distribution for X.

X	$p(x)$
0	$\binom{5}{3}/\binom{15}{3}$
1	$\binom{5}{2}\binom{10}{1}/\binom{15}{3}$
2	$\binom{5}{1}\binom{10}{2}/\binom{15}{3}$
3	$\binom{10}{3}/\binom{15}{3}$

5. Eighty-Five percent of the voters in a certain Texas county are members of the Tea Party. A survey of twenty randomly chosen voters from this county is conducted. Determine the **exact numeric values** of the following:
   a. The expected number of voters from the survey who are Tea Party members.
   b. The probability that at least two of the surveyed voters are NOT Tea Party members.

$X = \#$ Tea Party $\sim Bin(20, .85)$

a) $\mu = np = 20 \times .85 = 17$

b) $P(X \le 18) = 1 - P(X=19 \cup X=20)$

$= 1 - \left(\binom{20}{19} .85^{19} \cdot .15^{1} + \binom{20}{20} .85^{20} \cdot .15^{0}\right)$

$= 1 - (.136798 + .03876)$

$= .82442$

6. State Farm offers annual health insurance to married couples with maximum coverage as follows: $50000 per year if only the husband is sick, $30000 per year if just the wife is sick, and $90000 if both wife and husband get sick. Suppose that the husband has a 10% chance of getting sick and the wife has a 1% chance, and that these probabilities are independent of one another. What should a couple expect to pay for this insurance coverage?

X	$P(x)$	
0	$.9 \times .99$	$= .891$
30000	$.01 \times .90$	$= .009$
50000	$.99 \times .1$	$= .099$
90000	$.1 \times .01$	$= .001$

$$\mu = 30000 \times .009 + 50000 \times .099 + 90000 \times .001$$

$$= 5310$$

So, expect to pay $5310

7. College students average 6 hours of sleep per night with a standard deviation of 0.5 hours. Out of 1000 students, approximately how many of them will sleep between 5.25 and 6.75 hours?

(Chebychev)

$\mu = 6 \qquad \sigma = .5$

$$P(5.25 < X < 6.75) = P(\mu - .75 < X < \mu + .75)$$

$$\geq 1 - \frac{.5^2}{.75^2}$$

$$= \frac{5}{9} = .556$$

So, approx 556 students.

8. The lifespan of an HTC cellphone is distributed as X ~ N(18, 4), where X is measured in months. Find the probability that a randomly chosen phone lasts:
   a. more than 16 months.
   b. between 15 and 17 months.

a) $P(X > 16) = P\left(Z > \frac{16-18}{2}\right)$
$= P(Z > -1)$
$= 1 - .1587$
$= .8413$

b) $P(15 < X < 17) = P\left(\frac{15-18}{2} < Z < \frac{17-18}{2}\right)$

$= P(-1.5 < Z < -0.5)$

$= .3085 - .0668 = .2417$

9. Every time the Argentine soccer star *Sergio Aguer* plays a match, his chance of scoring a goal is 30%. Suppose he plays 70 matches. **Approximate** the following probabilities, leaving your answers in decimal form:
   a. Sergio scores more than 20.
   b. Sergio scores between 18 and 22 goals, inclusive.

$X \sim Bin(70, .3)$ $\mu = 21$, $\sigma^2 = 14.7$

$\sigma = 3.83$

a) $P(X > 20) = P(X \geq 21) = P(X > 20.5)$

$\approx P\left(Z > \frac{20.5 - 21}{3.83}\right) = P(Z > -0.1305)$

$= 1 - .4404 = .5596$

b) $P(18 \leq X \leq 22) = P(17.5 < X < 22.5)$

$= P(-.913 < Z < .391)$

$= .6554 - .1841 = .4713$

10. BONUS QUESTION – 8 points
Adohr Farms sells bottles of milk that are guaranteed to stay fresh for up to 15 days, with the chance of spoilage during this period being 4%. A consumer wants to buy enough milk to be sure to have at least a 99.9% chance of having at least two good bottles of milk on hand at the end of day 15. How many bottles should he buy?

X = # good bottles $\sim Bin(n, .96)$

$n = ?$

$P(X \geq 2) \geq .999$

$1 - P(X \leq 1) \geq .999$

$P(X \leq 1) \leq .001$

$P(X = 0) + P(X = 1) \leq .001$

$\binom{n}{0} .96^0 .04^n + \binom{n}{1} .96^1 .04^{n-1} \leq .001$

n	$P(X \leq 1)$
2	.0784
3	.004672
4	.000248
5	

$\leftarrow$ 4 Bottles !!