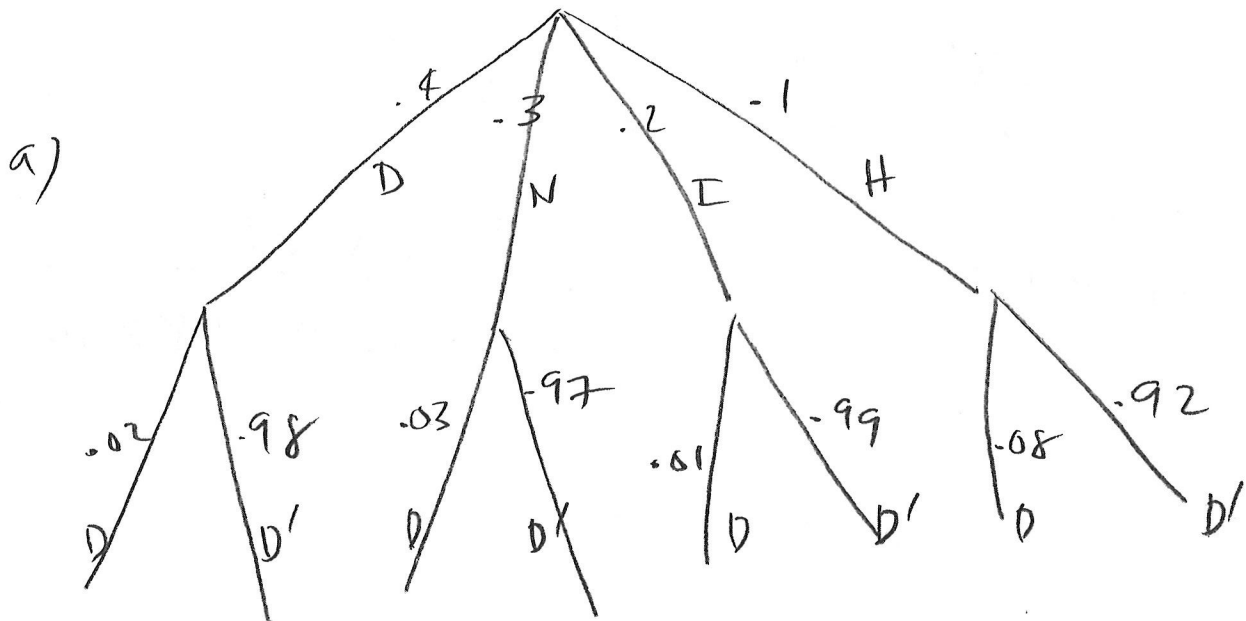


PINK

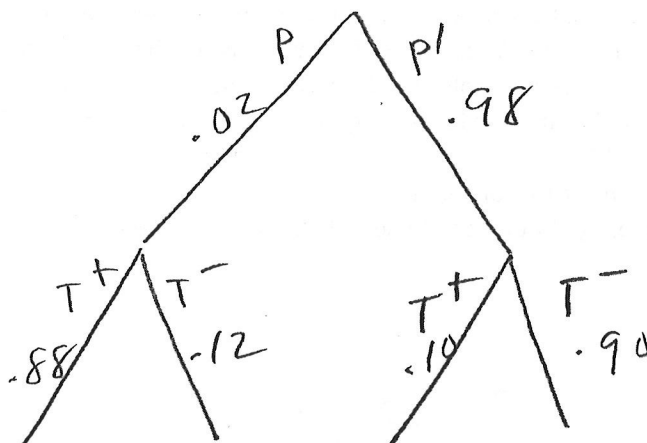
1. The quality control division of GM is interested in analyzing the quality of their cars as manufactured in four locations: Detroit, Nashville, Indianapolis, and Houston. They discover that 2% of the cars manufactured in Detroit are defective, 3% of those made in Nashville are defective, 1% of the cars from Indianapolis are defective, and 8% of the Houston cars are defective. GM produces 40% of its cars in Detroit, 30% in Nashville, 20% in Indianapolis, and 10% in Houston.
- Draw a tree diagram for this problem.
  - What is the probability that a car, chosen at random, will be free of defects?



b)

$$P(D) = .4 \times .98 + .3 \times .97 + .2 \times .99 + .1 \times .92$$
$$= .973$$

2. A young lady wishes to determine whether she is pregnant, so she buys a *ClearBlue* pregnancy test kit at her local drugstore. The kit is 88% accurate if the young lady is pregnant, and 90% accurate if they are not pregnant. Assume that generally, such a young lady is 2% likely to be pregnant. Suppose that this young lady administers her *ClearBlue* test and it indicates that she is pregnant. What is the probability that she actually is pregnant?



$$P(P | T^+) = \frac{.02 \times .88}{.02 \times .88 + .98 \times .10}$$

$$= .152249$$



4. An urn contains 12 red marbles and 10 black marbles. Three marbles are drawn, without replacement. Let  $X$  be the number of red marbles drawn. Find the probability distribution for  $X$ .

$X$	$p(x)$
0	$\frac{\binom{10}{3}}{\binom{22}{3}}$
1	$\frac{\binom{12}{1}\binom{10}{2}}{\binom{22}{3}}$
2	$\frac{\binom{12}{2}\binom{10}{1}}{\binom{22}{3}}$
3	$\frac{\binom{12}{3}}{\binom{22}{3}}$

5. A poll shows that 60% percent of the voters in a certain Texas county are members of the Tea Party. A survey of fifteen randomly chosen voters from this county is conducted. Determine the **exact numeric values** of the following:
- The expected number of voters from the survey who are Tea Party members.
  - The probability that at least two of the surveyed voters are Tea Party members.

$$X \sim \text{Bin}(15, .6)$$

$$a) \quad \mu = 15 \times .6 = 9$$

$$b) \quad P(X \geq 2) = 1 - \left( \binom{15}{0} \cdot .6^0 \cdot .4^{15} + \binom{15}{1} \cdot .6^1 \cdot .4^{14} \right)$$
$$= .999975$$

6. State Farm offers annual health insurance to married couples with maximum coverage as follows: \$50000 per year if only the husband is sick, \$30000 per year if just the wife is sick, and \$70000 if both wife and husband get sick. Suppose that the husband has a 5% chance of getting sick, while the wife has a 3% chance, and that these probabilities are independent of one another. What should a couple expect to pay for this insurance coverage?

$X$	$P(x)$
0	$.95 \times .97$
30000	$.03 \times .95$
50000	$.05 \times .97$
70000	$.03 \times .05$

$$\mu = 3385, \text{ so pay } \$3385$$

7. College students average 5 hours of sleep per night with a standard deviation of 0.2 hours. Out of 1000 students, approximately how many of them will sleep between 4.4 and 5.6 hours?

(Chebyshev)

$$P(4.4 < X < 5.6) = P(\mu - .6 < X < \mu + .6)$$
$$\geq 1 - \frac{(.2)^2}{(.6)^2} = \frac{8}{9} = .889$$

So 889 students.

8. The lifespan of a set of tires is distributed as  $X \sim N(28, 4)$ , where  $X$  is measured in months. Find the probability that a randomly chosen set of tires lasts:
- less than 26 months.
  - between 25 and 27 months.

a) .8413

b) .2417

9. Every time Susan attends mathematics class, she has a 20% chance of falling asleep. Suppose she attends 30 classes. **Approximate** the following probabilities, leaving your answers in decimal form:

- a. Susan falls asleep more than 5 times.  
 b. Susan falls asleep between 7 and 9 times, inclusive.

$$X \sim \text{Bin}(30, .2)$$

$$\mu = 6, \sigma^2 = 4.8$$

$$\sigma = 2.19$$

$$\begin{aligned} \text{a) } P(X > 5) &= P(X \geq 6) = P(X > 5.5) \\ &= P\left(Z > \frac{5.5 - 6}{2.19}\right) = P(Z > -0.2283) \\ &= 1 - .4207 = .5793 \end{aligned}$$

$$\begin{aligned} \text{b) } P(7 \leq X \leq 9) \\ &= P(6.5 < X < 9.5) \\ &= P\left(\frac{6.5 - 6}{2.19} < Z < \frac{9.5 - 6}{2.19}\right) \\ &= P(0.228 < Z < 1.5981) \\ &= .3659 \end{aligned}$$

10. BONUS QUESTION – 8 points

Adohr Farms sells bottles of milk that are guaranteed to stay fresh for up to 15 days, with the chance of spoilage during this period being 4%. A consumer wants to buy enough milk to be sure to have at least a 99.9% chance of having at least two good bottles of milk on hand at the end of day 15. How many bottles should he buy?

$$X = \# \text{ good bottles, } \sim \text{Bin}(n, .96)$$

$$n = ?$$

$$P(X \geq 2) \geq .999$$

$$1 - P(X \leq 1) \geq .999$$

$$P(X \leq 1) \leq .001$$

$$P(X=0) + P(X=1) \leq .001$$

$$\binom{n}{0} .96^0 .04^n + \binom{n}{1} .96^1 .04^{n-1} \leq .001$$

$n$	$P(X \leq 1)$
2	.0784
3	.004672
4	.000248 ← 4 Bottles !!
5	