Math 151
Test 1
February 6, 2017

CWID: KEY

1. Evaluate $\displaystyle\int \frac{(\ln x)^3}{x}\,dx = \int u^3\,du = \frac{1}{4}u^4 + C$

$u = \ln x \quad du = \frac{dx}{x}$

$= \frac{1}{4}(\ln x)^4 + C$

2. Evaluate $\int x^2 e^x \, dx$

$u = x^2 \quad du = 2x\,dx$

$dv = e^x dx \quad v = e^x$

$= x^2 e^x - 2\int x e^x dx \quad ; \quad U = x \quad dU = dx$

$dV = e^x dx \quad V = e^x$

$= x^2 e^x - 2x e^x + 2 \int e^x dx$

$= x^2 e^x - 2x e^x + 2e^x + C$

3. Evaluate $\int$ **$\cos^2(6x)\,dx$** $= \frac{1}{2}\int (\cos 12x + 1)\,dx$

$\cos^2\theta = \frac{1}{2}(\cos 2\theta + 1)$

$= \frac{1}{2}\left(\frac{\sin 12x}{12} + x\right) + C$

4. Find the minimum n for the midpoint rule so that the error in evaluating the following integral is < 0.001

$$\int_0^1 (1 + x^2)^3 \, dx$$

$$\frac{K(1-0)^3}{24n^2} < .001$$

$$K = \max\left(\left((1+x^2)^3\right)''\right)$$

$$= \max 6(5x^4 + 6x^2 + 1) = 72$$

$$\frac{72}{24 \cdot .001} < n^2 \qquad n > \sqrt{\frac{3}{.001}} = 54.7$$

$$n \geq 55$$

5. Determine whether the following integral converges. Show your work.

$$\int_3^4 \frac{dx}{(x-3)^{1/2}} = \lim_{a \to 3^+} \int_a^4 \frac{dx}{(x-3)^{1/2}} = \lim_{a \to 3^+} \left. \frac{(x-3)^{\frac{1}{2}}}{\frac{1}{2}} \right|_a^4$$

$= 2$. Converges.

6. Find the decimal value of the area bounded by **$y = x^2$ and $y = 2x - x^2$**

$$A = \int_0^1 (2x - x^2 - x^2)\,dx = \left(x^2 - \frac{2x^3}{3}\right)\Big|_0^1$$

$$= 1 - \frac{2}{3} = \frac{1}{3} = 0.\overline{3}$$

7. Set up - do not evaluate - the integral for finding the volume obtained by rotating the area between **x - y = 1 and y = x²-4x+3** about the line **y = 3.**

$y = x - 1 = x^2 - 4x + 3$

$0 = x^2 - 5x + 4$

$V = \int_1^4 \left(\pi(x-1-3)^2 - \pi(x^2-4x+3-3)^2\right) dx$

8. Evaluate $\int (1-x^2)^{1/2}\, dx$

$x = \sin\theta \qquad dx = \cos\theta \, d\theta$

$= \int \sqrt{1-\sin^2\theta}\ \cos\theta \, d\theta = \int \cos^2\theta \, d\theta$

$= \frac{1}{2} \int (\cos 2\theta + 1)\, d\theta$

$= \frac{1}{4} \sin 2\theta + \frac{1}{2}\theta + C$

$= \frac{1}{2} (\sin\theta \cos\theta + \theta) + C$

$= \frac{1}{2} \left(x \cdot \sqrt{1-x^2} + \arcsin(x) \right) + C$