Math 151
Test 1
February 6, 2017

CWID: KEY

1. Evaluate $$\int \frac{x+1}{(x^2+2x)^4}\,dx$$

$u = x^2+2x \qquad du = 2(x+1)\,dx$

$$\int \frac{\frac{1}{2}\,du}{u^4} = -\frac{1}{6}u^{-3} + C$$

$$= -\frac{1}{6}(x^2+2x)^{-3} + C$$

2. Evaluate $\int (\ln x)^2 \, dx$

$u = (\ln x)^2 \quad dv = dx$

$du = 2 \frac{\ln x}{x} dx \quad v = x$

$= x(\ln x)^2 - 2 \underbrace{\int \ln x \, dx}$

$U = \ln x \quad dV = dx$

$dU = \frac{dx}{x} \quad V = x$

$= x(\ln x)^2 - 2\left(x \ln x - \int dx\right)$

$= x(\ln x)^2 - 2x\ln x + 2x + C$

3. Evaluate $\int \mathbf{tan^3x\ sec^3x\ dx} = \int (\sec x \tan x) \sec^2 x \tan^2 x\, dx$

$$= \int \sec^2 x(\sec^2 x - 1) \sec x \tan x\, dx$$

let $u = \sec x \quad du = \sec x \tan x\, dx$

$$= \int u^2(u^2-1)\, du = \frac{u^5}{5} - \frac{u^3}{3} + c$$

$$= \frac{\sec^5 x}{5} - \frac{\sec^3 x}{3} + C$$

4. Find the minimum n for the midpoint rule so that the error in evaluating the following integral is < 0.001

$$\int_0^1 \mathbf{(1 + x^3)^4\, dx}$$

$$\frac{K\,(1-0)^3}{24n^2} < .001$$

$$K = \max\left(\left((1+x^3)^4\right)''\right)$$

$$\left((1+x^3)^4\right)'' = 12x(1+x^3)^2(2+11x^3)$$

$$\leq 12\,(4)\,(13) = 624$$

so $$n^2 > \frac{624}{(.001)24} = 26000$$

$$n > 161.24$$

$$n \geq 162$$

5. Determine whether the following integral converges. Show your work. $\int_1^\infty \mathbf{x\ e^{-x}\ dx}$ $= \lim_{a \to \infty} \int_1^a x e^{-x} dx$

$u = x \quad dv = e^{-x} dx$

$du = dx \quad v = -e^{-x}$

$= \lim_{a \to \infty} \left(-xe^{-x} \Big|_1^a + \int_1^a e^{-x} dx \right)$

$= \lim_{a \to \infty} \left(\frac{-x}{e^x} \Big|_1^a - e^{-x} \Big|_1^a \right)$

L'Hopital

$= \lim_{a \to \infty} \left(\frac{-1}{e^x} \Big|_1^a - e^{-x} \Big|_1^a \right)$

$= 0 + \frac{1}{e} - 0 + \frac{1}{e}$

Converges

6. Find the decimal value of the area bounded by **$y = 2x^2$ and $y = 4 + x^2$**

$$\left| \int_{-2}^{2} (2x^2 - 4 - x^2) \right| = \left| \left(\frac{x^3}{3} - 4x\right)\Big|_{-2}^{2} \right|$$

$$= 10.\overline{6}$$

7. Set up - do not evaluate - the integral for finding the volume obtained by rotating the area between **$x - y = 1$ and $y = x^2-3x-6$** about the line **$y = -2$.**

$$y = x-1 = x^2-3x-6$$

$$0 = x^2-4x-5$$

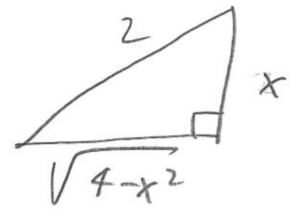
$$V = \pi\int_{-1}^{5} \left((x-1+2)^2 - (x^2-3x-6+2)^2\right)dx$$

8. Evaluate $\int (4 - x^2)^{1/2}\, dx$

$x = 2\sin\theta \qquad dx = 2\cos\theta\, d\theta$

$$\int (2\cos\theta)^2\, d\theta = 4\cdot\frac{1}{2}\int (\cos 2\theta + 1)\, d\theta$$

$$= 2\left(\frac{\sin 2\theta}{2} + \theta\right) + C$$



$$= 2\left(\sin\theta\cos\theta + \theta\right) + C$$

$$= 2\left(\frac{x}{2}\,\frac{\sqrt{4-x^2}}{2} + \arcsin\left(\frac{x}{2}\right)\right) + C$$