Math 151
Test 3
4.4.17

CWID: KEY

SHOW ALL YOUR WORK

1. **T/F**
   a. **If the limit comparison test results in a limit of 0, then both series converge.** F
   b. **If $a_n \to 0$, then Σa_n converges.** F
   c. **If the ratio test limit is 1, then the series diverges.** F
   d. **If Σa_n converges absolutely, then $\Sigma (-1)^{(n+1)} a_n$ converges.** T

2. Determine the limit of each of the following sequences:

a. $a_n = n!/(1/3)^n$

b. $a_n = (1+4/n)^n$

a. ∞

b. e^4

3. **Find x so that $\Sigma(\ln x)^n$ converges.**

geometric – so we need $-1 < \ln x < 1$

or $\frac{1}{e} < x < e$

4. If the nth partial sum of the series Σa_n is $s_n = (n-1)/(2n+1)$, find a_n.

$$a_n = S_n - S_{n-1} = \frac{n-1}{2n+1} - \frac{n-2}{2n-1}$$

$$= \frac{(n-1)(2n-1) - (n-2)(2n+1)}{4n^2-1}$$

$$= \frac{3}{4n^2-1}$$

5. **Determine whether the following series converges:** Σ **n/(2n² + n -1)**

Diverges

limit comparison

$$\lim_{n\to\infty} \frac{\frac{n}{2n^2+n-1}}{\frac{1}{n}} = \lim_{n\to\infty} \frac{n^2}{2n^2+n-1}$$

$$= \frac{1}{2}$$

so both diverge.

6. Use the integral test to determine the values of p so that the following series converges: Σ **(ln n)^p/n**

$$\int_1^\infty \frac{(\ln x)^p}{x}\,dx \qquad u = \ln x \qquad du = \frac{dx}{x}$$

$$= \int_0^\infty u^p\,du = \left.\frac{u^{p+1}}{p+1}\right|_0^\infty$$

converges if $p+1 < 0$

so $p < -1$

7. **For the following convergent series determine the minimum value of n so that S_n is accurate to within 0.0001.** $\Sigma\ (-1)^n/(n\ 2^n)$

$$.0001 = \frac{1}{(n+1)2^{n+1}}$$

So $n = 9$

8. Determine whether the following series converges absolutely $\Sigma\, n/(-2)^n$

Ratio Test $\lim_{n\to\infty} \frac{n+1}{2^{n+1}} \cdot \frac{2^n}{n}$

$= \frac{1}{2} < 1$

So converges absolutely. //