Math 151
Test 3
4.4.17

CWID: KEY

SHOW ALL YOUR WORK

1. T/F
   a. If a_n --> 0, then Σ a_n converges. F
   b. If Σ a_n converges absolutely, then Σ ($1/a_n$) diverges. T
   c. If the ratio test limit is 1, then the series diverges. F
   d. If the limit comparison test results in a limit of 0, then both series converge. F

2. Determine the limit of each of the following sequences:

a. $a_n = n!/(-2)^n$

b. $a_n = (1-2/n)^n$

a. $\pm\infty$ / diverges

b. e^{-2}

3. **Sum the following series $\sum_{1}^{\infty}$ ($3^n/(-4)^{n-1}$).**

$$= \frac{3}{(-4)^0} + \frac{3^2}{(-4)^1} + \frac{3^3}{(-4)^2} + \cdots$$

$$= 3\left(1 - \frac{3}{4} + \left(\frac{3}{4}\right)^2 - \left(\frac{3}{4}\right)^3 + \cdots\right)$$

$r = -\frac{3}{4}$

geometric

$$= 3\,\frac{1}{1+\frac{3}{4}} = \frac{12}{7}$$

$$= 1.714\ldots$$

4. If the nth partial sum of the series Σa_n is $s_n = (n-1)/(n+1)$, find a_n.

$$S_n - S_{n-1} = a_n$$

$$\text{So } a_n = \frac{n-1}{n+1} - \frac{n-2}{n} = \frac{n^2-n-(n^2-n-2)}{n(n+1)}$$

$$= \frac{2}{n(n+1)}$$

5. **Determine whether the following series converges:** $\Sigma\ n/(2n^2 + 3n + 1)^2$

Converges

limit comparison : $\lim_{n\to\infty} \dfrac{\dfrac{n}{(2n^2+3n+1)^2}}{\dfrac{1}{n^3}}$

$= \lim_{n\to\infty} \dfrac{n^4}{(2n^2+3n+1)^2} = \dfrac{1}{4},$

So $\underline{\text{both}}$ $\underset{2}{\text{converge}}$.

6. Use the integral test to find the values of p so that the following series converges:

Σ 1/(n (ln n)^p)

$$\int_2^\infty \frac{1}{x(\ln x)^p}\,dx \qquad u = \ln x \qquad du = \frac{dx}{x}$$

$$= \int_{\ln 2}^{\infty} u^{-p}\,du = \left.\frac{u^{-p+1}}{-p+1}\right|_{\ln 2}^{\infty}$$

converges if $-p+1 < 0$ so $p > 1$

7. **For the following convergent series, determine the minimum value of n so that S_n is accurate to within 0.0001.** $\Sigma\ (-1)^n/(n^2\ 3^n)$

Need $0.0001 = \dfrac{1}{(n+1)^2\ 3^{n+1}}$

So $n = 5$

8. **Determine whether the following series converges absolutely** $\Sigma\, n(-1/2)^n$

Ratio Test

$$\lim_{n\to\infty} \left| \frac{(n+1)\left(\frac{-1}{2}\right)^{n+1}}{n\left(\frac{-1}{2}\right)^{n}} \right|$$

$$= \lim_{n\to\infty} \frac{n+1}{n} \cdot \frac{1}{2} = \frac{1}{2} < 1$$

CONVERGES

ABSOLUTELY