Math 151
Test 2
March 10, 2017

CWID: KEY

1. SET UP the integral to determine the length of the parametric curve $x = e^{-t}$ and $y = e^{t}$, if $0 < t < 1$. DO NOT EVALUATE.

$$L = \int_0^1 \sqrt{(-e^{-t})^2 + (e^{t})^2}\, dt$$

$$= \int_0^1 \sqrt{e^{-2t} + e^{2t}}\, dt$$

2. The curve $y = x^3$ is revolved around the x-axis. Find the surface area of revolution when $0 < x < 1$

$$SA = \int_0^1 2\pi x^3 \sqrt{1 + (3x^2)^2}\, dx$$

let $u = 1 + 9x^4$

$du = 36x^3 dx$

x	u
0	1
1	10

$$SA = \int_1^{10} \frac{2\pi}{36} \sqrt{u}\, du = \frac{\pi}{18} \frac{u^{3/2}}{3/2} \Big|_1^{10}$$

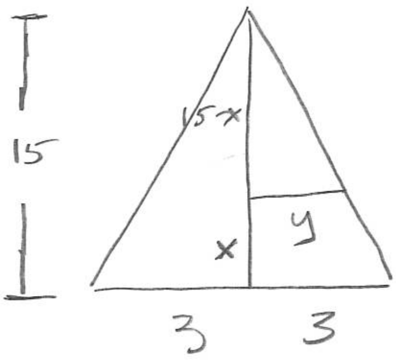
$$= \frac{\pi}{27} \left(10^{3/2} - 1^{3/2}\right) = 3.563$$

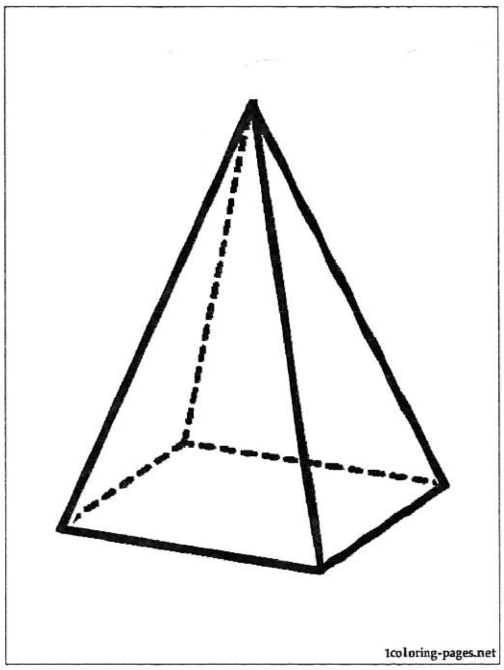
3. **Find the average value of f(x) = $3x^2$ on [0,2]**

$$\text{Ave} = \frac{1}{2}\int_0^2 3x^2\,dx = \frac{1}{2}\,x^3\Big|_0^2$$

$$= 4$$

4. **Set up (do NOT evaluate) the integral to do the following problem. The pyramidal tank, pictured, is full of water. Assume the base of the pyramid is a 6 ft. by 6 ft. square and that the pyramid's height is 15 ft. Recall that one cubic foot of water weighs 62.5 lb. How much work is required to pump all of the water out of the top of the tank?**

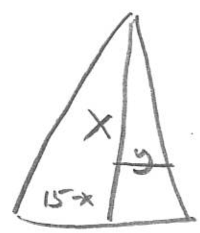




$$\frac{15-x}{y} = \frac{15}{3}$$

$$y = \frac{15-x}{5}$$

$$W = \int_0^{15} 62.5 \left(\frac{2(15-x)}{5}\right)^2 \cdot (15-x)\, dx$$



$$\frac{x}{y} = \frac{15}{3}$$

$$y = \frac{x}{5}$$

or

$$W = \int_0^{15} 62.5 \left(\frac{2x}{5}\right)^2 x\, dx$$

μ σ^2

5. **The IQ of the residents of Orange County is normally distributed as N(110, 100). Find P(90 < IQ < 120).** $= P\left(\frac{90-110}{10} < Z < \frac{120-110}{10}\right)$

$= P(-2 < Z < 1)$

$= 0.8413 - 0.0228$

$= 0.8185$

6. **Find all values of c so that $y = e^{cx}$ is a solution of $y'' + y' - 2y = 0$.**

$(c^2 + c - 2)e^{cx} = 0$

$(c+2)(c-1) = 0$

$c = -2$ or 1

7. **A 10 gallon barrel of diet coke is 50% food coloring. A mixture of diet coke that is 20% food coloring is poured into the barrel at a rate of 1 gallon per minute, so that the barrel is always perfectly mixed. If the barrel is being emptied at a rate of 1 gallon per minute, determine the number of gallons of food coloring that are in the barrel after 10 minutes.**

$$y' = .2 - \frac{y}{10} = \frac{2-y}{10}$$

$$y(0) = 5$$

$$\int \frac{dy}{2-y} = \int \frac{dt}{10} \quad \text{or}$$

$$-\ln|2-y| = \frac{t}{10} + c, \quad -\ln 3 = c$$

So - now let $t=10$; note $y > 2$ because y starts at 5 & $\rightarrow 2$.

$$-\ln|2-y| = 1 - \ln 3$$

$$\ln \frac{3}{|2-y|} = 1, \quad \text{so} \quad \frac{3}{|2-y|} = e$$

$$|2-y| = \frac{3}{e} \qquad y = 2 + \frac{3}{e} = 3.1$$

8. **I bought a 180° cup of coffee at Coffee Bean and Tea Leaf yesterday morning and sat in the 80° shop reading my paper. At noon, the coffee had cooled to 160°. At 1:00 p.m. it had gone down to 140°. When did I buy my coffee?**

	t	T
	0	180
(NOON)	a	160
	$a+1$	140

$$T = 80 + 100e^{kt}$$

$$160 = 80 + 100e^{ak}$$

$$140 = 80 + 100e^{(a+1)k}$$

or $\quad .8 = e^{ak}$

$.6 = e^{ak} \cdot e^{k} \quad \longrightarrow \quad .75 = e^{k}$

$$k = \ln(.75)$$

$$.8 = e^{a(\ln .75)} = e^{\ln(.75^{a})}$$

$$.8 = .75^{a} \qquad a = \frac{\ln .8}{\ln .75} = .775 \text{ hours}$$

or coffee bought at 11:13:30