Math 151

Test 2

March 10, 2017

CWID: KEY

1. SET UP the integral to determine the length of the parametric curve x = t + cos(t), y = t - sin(t), if 0 < t < π. DO NOT EVALUATE.

$$L = \int_0^{\pi} \sqrt{(1-\sin t)^2 + (1-\cos t)^2}\, dt$$

2. **The curve y = x^3 is revolved around the x-axis. Find the surface area of revolution when -1 < x < 0.**

to make radius positive

$$SA = \int_{-1}^{0} 2\pi(-x^3)\sqrt{1+(-3x^2)^2}\,dx$$

$$= \int_{-1}^{0} -2\pi x^3 \sqrt{1+9x^4}\,dx$$

$u = 1+9x^4$, $du = 36x^3 dx$

x	-1	0
u	10	1

$$= \int_{10}^{1} -\frac{2\pi}{36}\sqrt{u}\,du$$

$$= -\frac{\pi}{18}\frac{u^{3/2}}{3/2}\Big|_{10}^{1} = \frac{\pi}{27}u^{3/2}\Big|_{1}^{10}$$

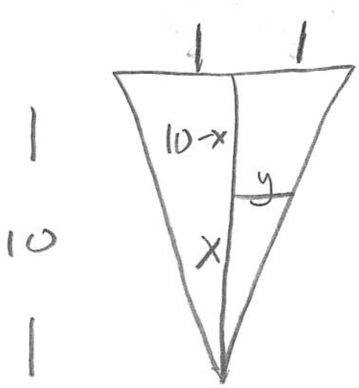
$$= \frac{\pi}{27}\left(10^{3/2}-1\right) = 3.563$$

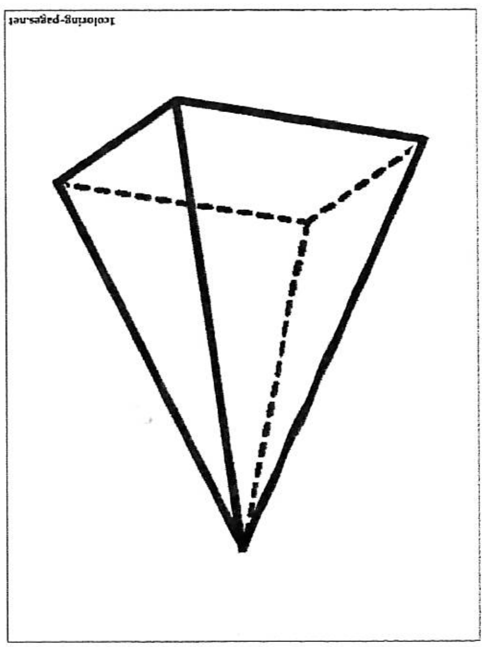
3. **Find the average value of f(x) = 3x² - 1 on [0,3]**

$$Ave = \frac{1}{3}\int_0^3 (3x^2-1)\,dx = \frac{1}{3}\left(x^3-x\right)\Big|_0^3$$

$$= \frac{1}{3}(24) = 8$$

4. Set up (do NOT evaluate) the integral to do the following problem. The inverted pyramidal tank, pictured, is full of water. Assume the base of the pyramid is a 2 ft. by 2 ft. square and that the pyramid's height is 10 ft. Recall that one cubic foot of water weighs 62.5 lb. How much work is required to pump all of the water out of the top of the tank?





$$\frac{y}{x} = \frac{1}{10}$$

$$y = \frac{x}{10}$$

$$W = \int_0^{10} 62.5 \left(\frac{2x}{10}\right)^2 (10-x)\, dx$$

$$\frac{10}{1} = \frac{10-x}{y}$$

or

$$W = \int_0^{10} 62.5 \left(\frac{2}{10}(10-x)\right)^2 \cdot x\, dx$$

$$y = \frac{10-x}{10}$$

μ σ^2

5. The IQ of the residents of Ventura County is distributed normally as N(100, 100). Find the 90th percentile IQ.

$$.9 = P(-\infty < IQ < X_{.9})$$

$$= P\left(-\infty < \frac{IQ - 100}{10} < \frac{X_{.9} - 100}{10}\right)$$

$$= P\left(-\infty < Z < \frac{X_{.9} - 100}{10}\right)$$

$$\frac{X_{.9} - 100}{10} = 1.3$$

$$X_{.9} = 113$$

6. **Find all values of c so that $y = e^{cx}$ is a solution of $2y'' + y' - y = 0$.**

$$e^{cx}(2c^2 + c - 1) = 0$$

$$(2c - 1)(c+1) = 0$$

$$c = \frac{1}{2}, -1$$

7. **A 5 gallon barrel of *milk* is 10% soy. A mixture of *milk* that is 20% soy is poured into the barrel at a rate of 1 gallon per minute, so that the barrel is always perfectly mixed. If the barrel is being emptied at a rate of 1 gallon per minute, determine the number of gallons of soy in the barrel after 5 minutes.**

$y(0) = .5$

$y' = .2 - \frac{y}{5} = \frac{1-y}{5}$

$\frac{dy}{1-y} = \frac{dt}{5}$, $\quad -\ln|1-y| = \frac{t}{5} + c$

$-\ln|1-.5| = c \qquad c = -\ln\frac{1}{2} = \ln 2$

Now, let $t=5$:

$-\ln|1-y| = 1 + \ln 2$ (y<1 because y starts at .5 & $\to$ 1)

$-1 = \ln|2(1-y)|$, $\quad 2(1-y) = \frac{1}{e}$

$y = 1 - \frac{1}{2e}$

$= 0.816$

8. **I bought a 150° cup of coffee at Coffee Bean and Tea Leaf yesterday morning and sat in the 60° shop reading my paper. At noon, the coffee had cooled to 130°. At 1:00 p.m. it had gone down to 90°. When did I buy my coffee?**

	t	T
	0	150
(noon)	a	130
	a+1	90

$$T = 60 + 90e^{kt}$$

$$130 = 60 + 90e^{ak}$$

$$90 = 60 + 90e^{(a+1)k}$$

$$\frac{7}{9} = e^{ak}, \quad \frac{3}{9} = e^{ak} \cdot e^{k}$$

$$e^{k} = \frac{3}{7} \qquad k = \ln\left(\frac{3}{7}\right),$$

$$\frac{7}{9} = e^{a\ln\left(\frac{3}{7}\right)} = e^{\ln\left(\left(\frac{3}{7}\right)^{a}\right)}$$

$$\frac{7}{9} = \left(\frac{3}{7}\right)^{a} \qquad a = \frac{\ln(7/9)}{\ln(3/7)}$$

$$= 0.29$$

Purchase at approx 11:42