Automata Theory

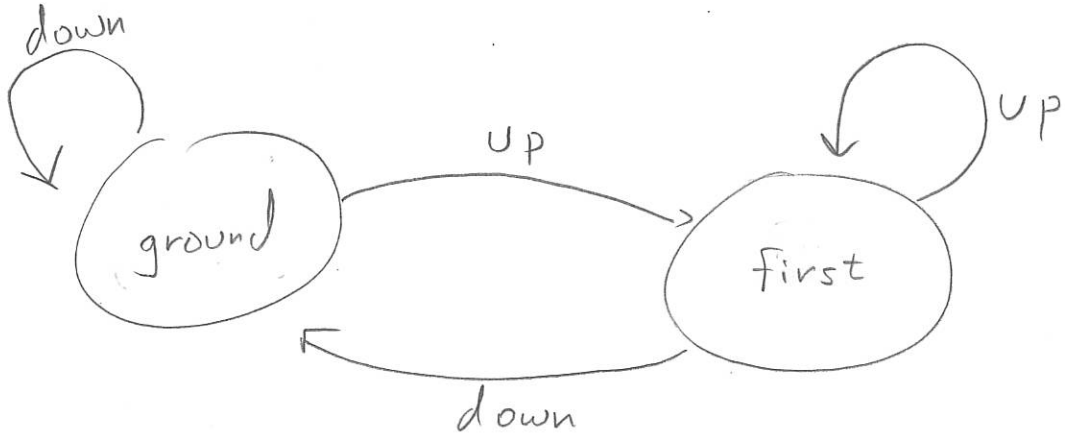
Test 1

2.27.20

CWID: KEY

1. Construct the DFA diagram to control an elevator in a 2 story building.

Assume there is an indicator light in the elevator for the current floor and a button on each floor controlling or calling the elevator up or down. The elevator can be at one of two floors: ground or first. Assume there are two buttons - up/down inside the elevator to select the destination.



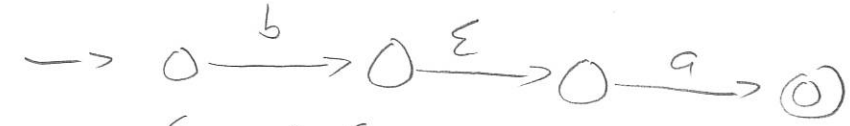
2. Construct an NFA diagram to recognize the language generated by (a ∪ b)* ∪ (ba)* ∪ ab

a

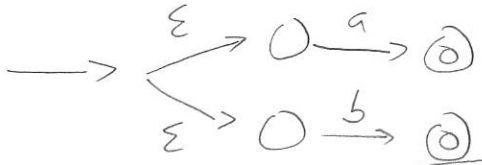
b

E: ab

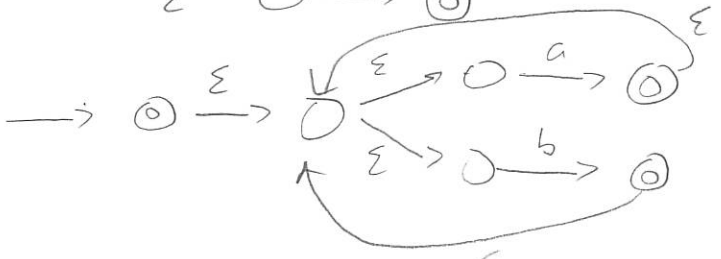
ba



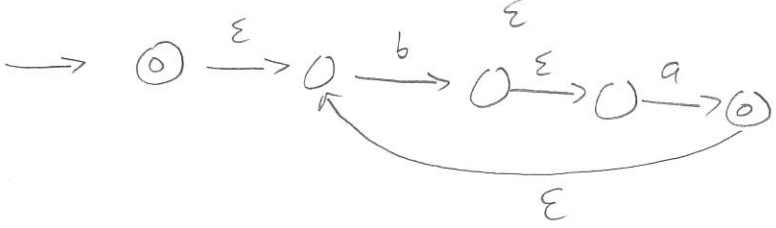
a ∪ b



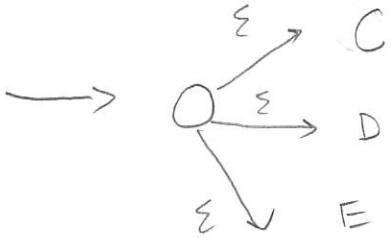
C: $(a \cup b)^*$



D: $(ba)^*$



Solution :



3. **Construct a CFG to generate a signed number of the form -454.8968 or 0.2573 or -473 or 821 (no + sign needed for positive numbers).**
   Here are some hints:
   S --> <number>
   <number> --> <sign> <part>
   <number> --> ????
   <part> --> ????
   etc.

S → <number>

<number> → <sign> <part>

<number> → <sign> <part> . <part>

<sign> → - | ε

<part> → <digit>

<part> → <digit> <part>

<digit> → 0 | 1 | 2 | 3 | ... | 9 | ε

4. **Trace the stack, input stream, and state sequence for the following CFG state diagram in Figure 2.17 on p. 116 if the input is aabcc.**

state	stack	input
q_1	ε	ε
q_2	$	ε
q_2	a $	a
q_2	a a $	a
q_5	a a $	ε
q_5	a a $	b
q_6	a a $	ε
q_6	a $	c
q_6	$	c
q_7	ϕ	ε

Accept

5. Construct a verbal description of the TM that will recognize {A#B: B^R is a substring of A, where A,B ∈ {a,b}*}. For example, the TM will recognize ababbb#bba, but not baabb#aabb.

General Idea: B^R is a substring of A iff B^R matches $A_1 A_2 \ldots A_k$ or $A_2 A_3 \ldots A_{k+1}$ ... or $A_{n-k+1} A_{n-k+2} \ldots A_n$ — character by character

where $|B^R| = k$, $|A| = n$

else reject.

So move right to left in B, comparing w/ $A_j \ldots A_{j+k-1}$ left to right etc $(j = 1 \ldots n-k+1)$ until there is a 1 to 1 match on B^R or all alignments fail, always doing comparisons character by character.

6. **If n unmarked coins consist of n-1 of equal weight and 1 that is odd (either heavy or light) and 4 weighings allow us to find and classify the odd coin, what is the maximum value of n? (DO NOT describe the steps of each weighing.)**

max = 40

4 weighings creates $H = 4 \log_2 3 = \log_2 81$

$\therefore \log_2 \lceil (\#\text{ coins}) \cdot 2 \rceil \leq \log_2 81$

So # coins ≤ 40, so that maximum n is 40.