THEOREM 4 *If $\{f(\mathbf{x} \mid \theta)\}$ has a monotone likelihood ratio in terms of a statistic T, the power function of the Neyman-Pearson critical region for testing θ_0 against θ_1, with $\theta_0 < \theta_1$ is an increasing function of θ.*

COROLLARY *If $\{f(\mathbf{x} \mid \theta)\}$ has a monotone likelihood ratio in terms of a statistic T, the Neyman-Pearson critical region for testing θ_0 against θ_1, where $\theta_0 < \theta_1$, is UMP for $\theta \le \theta^*$ against $\theta > \theta^*$.*

To prove the theorem we need to show that for any θ' and θ'', with $\theta' < \theta''$, the power of the Neyman-Pearson region C at θ'' is at least as great as its power at the point θ': $\pi_C(\theta'') \ge \pi_C(\theta')$. But we know that the inequality $L(\theta')/L(\theta'') < K$ is equivalent to an inequality of the form $T > K'$, which in turn is equivalent to $L(\theta_0)/L(\theta_1) < K''$ (for some constants K' and K''). This means that C is most powerful not only for θ_0 vs. θ_1, but also for θ' vs. θ''. Thus, for any test ϕ with no greater α (at θ'), the power of C at θ'' is at least as great as the power of ϕ at θ''. That this is true in particular for the test $\phi \equiv \alpha_C$ establishes the theorem:

$$\pi_C(\theta'') \ge \pi_\phi(\theta'') = E_{\theta''}(\phi) = \alpha_C = \pi_C(\theta').$$

To show the corollary we argue much as we did in Example 9.9c: The N-P region C for θ_0 vs. θ_1 is also most powerful for θ^* against any particular $\theta > \theta^*$ and is therefore UMP against the composite alternative $\theta > \theta^*$. But because the power function of C is monotone increasing, the largest "α" in $\theta \le \theta^*$ is the power at θ^*:

$$\alpha_C = \sup_{\theta \le \theta^*} \pi_C(\theta^*) = \pi_C(\theta^*).$$

The power of a test ϕ with $\alpha \le \alpha_C$ does not, in particular, exceed the power of C *at* θ^* and is therefore uniformly smaller than the power of C throughout $\theta > \theta^*$, as asserted in the corollary.

The force of the corollary is that when the family $\{f(x \mid \theta)\}$ has a monotone likelihood ratio, we can construct a family of UMP tests of $\theta \le \theta^*$ against $\theta > \theta^*$ as the Neyman-Pearson tests of θ_0 against θ_1 for arbitrarily chosen values θ_0 and θ_1 such that $\theta_0 < \theta_1$.