EXERCISE 8.1.

1. Suppose that X is a Poisson random variable with parameter λ. We take a sample $X_1, X_2, \ldots, X_n$ of X and want to test $H_0: \lambda = 2$ versus $H_1: \lambda > 2$. What do you feel would be a reasonable critical region?

2. Suppose that X is a normal random variable with unknown mean μ and variance 9. If we take a random sample of 16 observations of X and want to test $H_0: \mu = 2$, show that each of the following critical regions is of size .05 (that is, the probability of type I error is .05 for each).

(a) $\bar{X} > 3.23$ (b) $\bar{X} < .77$ (c) $1.952 < \bar{X} < 2.048$

For what type of alternative hypothesis, if any, would each of these be appropriate?

3. We might assume that weights of 4-month-old pigs are normally distributed with standard deviation $\sigma = 10$ pounds. A standard ration is known to produce an average weight of 120 pounds. If we desire to check whether a new ration is equally good, we might take a sample of 25 pigs, feed them the new ration, check their weights at age 4 months, and test $H_0: \mu = 120$ versus $H_1: \mu < 120$ using the critical region $\bar{X} < 116.08$. Compute the probability of type I error for this test and derive the power function. What is the power of the test if $\mu = 115$ pounds? (That is, what is $Q(115)$?)

4. Suppose that the number of ounces that a bottling machine puts in a coke bottle is a normal random variable with mean μ and variance σ^2. If σ^2 is too large, many bottles will be filled to overflowing; for the setting on the machine controlling the mean, a σ^2 not exceeding $\frac{1}{4}$ is tolerable. Suppose we take a sample of 20 bottles filled by the machine and want to test $H_0: \sigma^2 = \frac{1}{4}$ versus $H_1: \sigma^2 > \frac{1}{4}$. We decide to reject H_0 if

$$\sum_{i=1}^{20} (X_i - \bar{X})^2 > 7.536.$$

What is the probability of type I error for this test?

5. Assume that the time to failure (in hours) of a certain type of vacuum tube is an exponential random variable with parameter λ. We put 10 of these tubes on test and want to test the hypothesis $H_0: \lambda = .001$ versus $H_1: \lambda < .001$. We decide to wait only until the first tube fails and will reject H_0 if the time to this first failure is no larger than 5.129 hours. Show that the probability of type I error for this test is .05. (Thus we are concerned with the minimum sample value of a sample of size 10 of an exponential random variable; the reader might like to review Example 6.3.4 to see the density function for this minimum value.)

6. Assume that X is a uniform random variable on the interval from $-\theta$ to θ. We observe one value of X and want to test $H_0: \theta = 1$ versus $H_1: \theta > 1$; we decide to reject H_0 if the sample value exceeds .99. Compute the probability of type I error for this test and draw the power function. Can you suggest a better rule?

EXERCISE 8.2.

1. Suppose that X is a Bernoulli random variable with parameter p. We take a random sample of 4 observations of X and want to test $H_0:p = \frac{1}{4}$ versus $H_1:p = \frac{3}{4}$. If we reject H_0 only if we get 4 successes in the sample, compute the values of α and β.

2. Given that X is a uniform random variable on the interval $(0, \theta)$, we might test $H_0:\theta = 1$ versus the alternative $H_1:\theta_1 = 2$ by taking a sample of 2 observations of X and rejecting H_0 if $\bar{X} > .99$. Compute α and β for this test.

6. If X is a normal random variable with mean 0, derive the uniformly most powerful test of $H_0:\sigma^2 = 2$ versus $H_1:\sigma^2 > 2$ of size α, based on a random sample of n observations of X.

7. Derive the form of the Neyman-Pearson critical region for testing $H_0:\lambda = \lambda_0$ versus $H_1:\lambda = \lambda_1(\lambda_1 > \lambda_0)$, based on a random sample of n observations of an exponential random variable with parameter λ.

8. Assume that the probability of getting a head on a single flip of a given coin is p and derive the most powerful critical region for testing $H_0:p = \frac{1}{2}$ versus $H_1:p = \frac{3}{4}$, based on a sample of n flips of the coin.

9. Assume that X and Y are independent normal random variables, each with variance 1. Given a random sample of n observations of each, derive the most powerful critical region of size α for testing $H_0:\mu_X - \mu_Y = 0$ versus

$$H_1:\mu_X - \mu_Y = 1.$$

EXERCISE 8.3.

1. Assume that the annual rainfall at a certain recording station is a normal random variable with mean μ and standard deviation 2 inches. The rainfall recorded (in inches) in each of 5 years was 18.6, 20.4, 17.3, 15.1, and 22.6. Test the hypothesis that $\mu = 21$ versus the alternative $\mu < 21$ with $\alpha = .1$.

5. A producer of frozen fish is being investigated by the Bureau of Fair Trades. Each package of fish which this producer markets carries the claim that it contains 12 ounces of fish; a complaint has been registered that this claim is not true. The Bureau acquires 100 packages of fish marketed by this company and, letting x_i be the observed weight (in ounces) of the i-th package, $i = 1, 2, \ldots, 100$, they find

$$\sum x_i = 1150, \qquad \sum x_i^2 = 13{,}249.75.$$

It would seem reasonable to assume that the true weights of packages that they market are normally distributed with mean μ and variance σ^2, neither of which is known. With $\alpha = .01$, would the Bureau accept or reject $H_0\colon \mu = 12$ versus $H\colon \mu < 12$, based on this sample?

6. In deciding whether a certain type of plant would be appropriate for hedges, it is of some importance that individual plants exhibit small variability in the amounts they will grow in a year (at the same age). Specifically, we might assume that the growth made by a plant of a specific type and age (for given climatic conditions) is a normal random variable with mean μ and variance σ^2. Then, to decide whether the plant would be appropriate for hedges, we might like to test $H_0\colon \sigma^2 = \frac{1}{4}$ versus $H_1\colon \sigma^2 < \frac{1}{4}$ with $\alpha = .05$ (measurements made in feet). Suppose that we record the growth of 5 plants of this type for 1 year and find them to be 1.9, 1.1, 2.7, 1.6, and 2.0 feet. Should we accept H_0?

7. Derive the likelihood ratio test criterion critical region for testing the hypothesis that $\lambda = \lambda_0$ versus the alternative that $\lambda > \lambda_0$ where λ is the parameter of an exponential random variable. Assume that you have available a random sample of n observations of X and that you want your probability of type I error to be α.

8. The time to failure of a certain vacuum tube is known to be an exponential random variable with parameter λ when used in a particular type of circuit. A thousand of these tubes are placed in operation and the sum of their times to failure is 109,652 hours. With $\alpha = .05$, would you reject $H_0\colon \lambda = .008$ versus $H_1\colon \lambda > .008$ on the basis of this sample?

9. The number of accidents in an industrial plant per month is assumed to be a Poisson random variable with parameter λ. Given a sample of the number of accidents per month in a given year (thus $n = 12$), derive the likelihood ratio test criterion for testing $H_0\colon \lambda = \lambda_0$ versus $H_1\colon \lambda < \lambda_0$ for a fixed α. Suppose that a given industry finds that they had a total of 15 accidents last year. With $\alpha = .1$, should they reject $H_0\colon \lambda = 2$ versus $H_1\colon \lambda < 2$? (Use the result of Theorem 8.3.4.)

10. The number of times that an electric light switch can be turned on and off before it fails is a geometric random variable X with parameter p (thus X is the number of turnons until failure). Given a random sample of 10 switches, and that the sum of the number of turnons to failure (for all 10) was 15,169, would you reject $H_0\colon p = .00005$ versus $H_1\colon p > .00005$ with $\alpha = .05$? (Use Theorem 8.3.4.)